Since coming to BYU-Idaho in 2009, I have been excited about the beauty and simplicity of the Learning Model. For me, the three components of the Learning Model—prepare, teach one another, and ponder/prove—each brought their own challenges and opportunities. I quickly found that it was simple to incorporate teach one another and ponder/prove into my courses. However, I struggled to effectively create and implement the preparation element of the Learning Model. In attending a Brown Bag where faculty met with a panel of students, a question was asked regarding preparation assignments. The students had interesting responses to the question. One student's response that stands out to me still was, “Don't give us a preparation assignment and then teach the same exact information the next class. If you do this, then why am I doing the preparation?” As I moved forward in developing preparation assignments, I worked hard not to create preparation assignments that were a carbon copy of my lessons.

A few years later, I began working on my doctorate with Elaine Wagner and Heidi Turner, mathematics faculty members at BYU-Idaho. Our advisor Dr. David Yopp encouraged us to research the use of example generation in learning novel mathematical concepts. For our dissertation, we researched and created example generation tasks to use as preparation assignments in a first-semester calculus course. We collected data on how students learned novel mathematical concepts using example generation tasks. We continue to use these example generation tasks as preparation assignments to enrich our courses. Elaine and I also created a set of example generation tasks to use in second and third semester calculus classes.

Why Use Example Generation?
A common practice in mathematics classrooms (and I assume other classrooms) is the following: student sits at a desk, teacher stands at the chalkboard demonstrating how to complete problems, student copies those problems in his/her notebook, and student goes home and completes homework that is like the problems completed in class. From this viewpoint, students mimic the actions of the
teacher, and there is little to no preparation, teach one another, or ponder/prove. Using example generation tasks such as preparation assignments changes this viewpoint. Preparation assignments invite students to engage in pre-class activities and discussions such that they come prepared to share what they have learned with classmates and ask questions that further class discussions.

Example generation is the act in which learners generate, create, construct, or produce an example to expand their individual example space (Watson & Mason, 2005; Yopp, 2014). When students complete the example generation tasks to the best of their ability, the discussion in class is richer and deeper. Please note that students are not completing example generation tasks for correctness but completing each part to the best of their ability. When students do this, there will be times when you can move more quickly through the material. We saw this when students completed the Fubini task used in Calculus III. The discussion generated by the students was insightful and showed a conceptual understanding of the topic. In this experience, students came prepared, and taught the entire section to one another, which enabled me to guide them into more difficult concepts for that unit.

By using example generation tasks, you have a variety of examples, correct and incorrect, to use in teaching the concept. When using student work, the students become more engaged and excited that something they created is being used to learn the concept. The students’ active participation in generating examples promotes learning and mathematical reasoning skills (Orme, Wagner, & Turner, 2014).

How to use Example Generation?

Example generation will look different depending on content area and expectations, but there are a few guiding principles we should always keep in mind. The most important principle when using example generation is to ensure that the task is accessible to the students and uses foundational knowledge that they currently possess.

Example generation tasks can vary in type and usage: preparation assignments, projects based on material learned in class, and/or exam questions. Most of the example generation tasks we designed were preparation assignments to give students a jump-start on thinking through the new ideas presented in class. Students completed the tasks to the best of their ability and shared their ideas during class. Students continued to build knowledge based on what they learned in the tasks. We created a few example generation tasks as projects where students learned the concept first in class and then worked in groups outside of class to apply those concepts to a project. In Calculus I, students learn the concept of related rates in class and complete practice homework problems. Following their initial exposure to related rates, groups of students were asked to generate related rates examples based on given conditions. The students worked in groups to develop the problem and find the solution. They then presented their problem to the class. Each semester we receive positive feedback from the students: they like this task and find it useful in helping them learn the concept.

A third way to use example generation tasks is to do True/False questions on exams. The students decide whether the statement is true or false and then give examples to show a statement is true and counterexamples to show a statement is false.

In asking students to generate examples for more extensive or detailed questions, there are certain aspects each task should have: general instructions, a definition or theorem, a procedure or direction, further explorations, and metacognitive questions.

In the general instructions, it is beneficial to explain why the students are completing this task. When students understand the reasoning behind the task, they are more likely to try hard things. Be sure to include the goal or objective of the task. Students need to know what
The students’ active participation in generating examples promotes learning and mathematical reasoning skills.

they should learn or understand by the time they complete the task. General instructions must include student expectations.

The next part of your task will depend upon your goal. For our tasks we asked students to reason through mathematical theorems and/or definitions. Be sure to use the theorem/definition exactly as it is written in your curriculum material. These tasks can be a great asset in teaching students how to read content-specific language.

In the procedure or directions part of the task, you can add in more explanation that states the definition/Theorem in simpler terms. It is here where students begin to create their examples. As you develop a sequence of tasks, it is suggested that for the first few you provide structured guidance to help students learn how to create examples that will help them learn the material. After three or four tasks, students can begin to create their own examples with limited guidance, and eventually without any guidance at all.

As students embark in further explorations, you can add intermediate questions that ask students to consider specific parts of the theorem/definition. For example, you may ask them why an interval must be closed or why a function must be continuous. In this part of the task, ask students to generate another example, or even create a counterexample. By generating additional examples, students solidify their understanding of key concepts and compare and contrast between examples. In conducting further explorations, students may see connections that they did not see with only one example.

In the final section of the task, students should be asked metacognitive questions where they reflect on their thinking and what they have learned. The metacognitive questions can deepen in-class discussions. Students can share what they do not understand and bring those questions to the class discussion.

**Student Reflections on Example Generation**

Not all students will like doing example generation tasks and nothing you do will change that attitude. A student we identified as Randy said: “I didn't pay to teach myself concepts and do it wrong; that's not what school is about, a teacher teaches the material, then the student does the homework to solidify their understanding of it...The teaching style of teach yourself is crap, and in real life as an engineer, there will be a lot of people working on the same thing...you will never be given something new and told to figure it out all alone” (Orme, Wagner, & Turner, 2014).
When students complete the example generation tasks to the best of their ability, the discussion in class is richer and deeper. In all the data we collected, we only found a few students who expressed, at least to us, such strong feelings against the use of example generation.

Students will struggle with completing the example generation tasks and not always see the vision for completing the tasks in their learning. But this is more common at the beginning of the semester when the method is unfamiliar, and the students lack confidence. However, over time students become more confident in generating and using examples to learn the content. Common comments I hear at the beginning of the semester in my classes are the following:

• “I didn’t understand anything,”
• “this was frustrating,”
• “[the examples] were not useful, I got strange answers,” and
• “this was hard, I don’t think I did anything right.”

But, by the end of the semester many students will see benefits for completing the tasks. Students will also begin to generate their own examples, without being prompted, to better understand a concept. Common comments shared verbally and written were the following:

• “Even though it was hard, it pushes me to think,”
• “I did not always get it correct, but I learned how to think,”
• “By creating my own examples, I was able to see what worked and what did not work. I still am unsure of how to work with the third dimension, but I think I’m starting to get it,”
• “It is useful because the examples are more meaningful, and it allows me to personally understand it better and make more connections,”
• “It also forces you to troubleshoot your own problems and see why something does or does not work,” and
• “If we create our own examples, at least for me, it sticks better in my mind.”

Summary
How can you use example generation in your class? The answer is simple: If you create examples to help students learn a concept, ask the students to also create those examples. It can be as simple as asking students to create a graph of a function or create an example of a sentence with a dangling participle. Example generation can also be more detailed when students are asked to create examples to understand the important aspects of a theorem or definition or create an example of a chemical compound using specific atoms. Despite student frustrations, when they complete the preparation example generation tasks, they engage in learning the content and have more meaningful in-class discussions.

References