Negative Index of Refraction of Cross-bar Metamaterial

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ABSTRACT

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The purpose of this project is two-fold: (1) construct a metamaterial and (2) test that material for a negative index of refraction using the BYU-Idaho Physics Department’s x-band microwave optics equipment. For construction, the facilities of BYU-Idaho’s Electrical Engineering Department and Chemistry Department were utilized. The metamaterial design is built on FR4 printed circuit boards (PCBs) with a copper overlay on both sides. The copper on the PCB is a double cross-bar design repeated on both sides throughout each board. A variety of boards are built with different sizes to form a wedge of metamaterials. The microwave optics equipment is then used to transmit a microwave beam rated at 10.7 GHz, refract it with the metamaterial wedge, and detect the intensity over a spread of angles with the receiver and a LabVIEW program. A MATLAB program is then used to analyze the data and place it on two graphs. One graph depicts the voltage at the actual measured angles and is used to compare the results with measured background microwave radiation. The second depicts the voltage in terms of the refracted angles, the peak of the voltage telling us the angle of the refracted beam. According to Snell’s Law, a negative index of refraction can be verified with a negative refracted angle.
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Chapter 1

Introduction and Background

1.1 Project Background

The BYU-Idaho Physics Department has x-band microwave optics equipment available. A variety of projects have been done with this equipment ranging from atmospheric ice scattering to life detection of rats. The research project about which this thesis is written looks into the electromagnetic effects of metamaterials.

A metamaterial could be defined as a manmade material that has electromagnetic properties typically not found in nature. At the heart of metamaterials is the ability to control and manipulate the permittivity and permeability of an object. Permittivity and permeability are fundamental electromagnetic properties and will be discussed later. Such influence allows for materials to be made with certain and specific desired electromagnetic properties that would be otherwise unattainable. This has opened up new and exciting areas to consider in the realm of physics. This includes but is not limited to: a negative index of refraction, reversal of the Doppler effect [1], reversal of Vavilov-Cerenkov effect [1], evanescent field growth in amplitude (leading to exciting potential in what has been named perfect imaging and superlenses) [2], and cloaking and invisibility [3]. For the purposes of this project, focus will be placed upon the negative index of refraction of a metamaterial.

The purpose of this research is two-fold. The first is to actually construct a metamaterial using a previously determined design. This includes the construction of the actual metamaterial pattern itself
and also the preparation of the necessary materials to aid in testing. The second purpose is to test the constructed metamaterial and analyze the results. The design in question uses printed circuit boards (PCBs) that have metallic (copper) cross-bar inclusions throughout both sides of each PCB [4].

1.2 Theory

1.2.1 Electromagnetic Radiation

To begin to understand why metamaterials work, it is necessary to visit the topic of electromagnetic radiation. Eugene Hecht gives a description sufficient for our purposes and the following process can be found therein [5]. Regardless of where in the electromagnetic spectrum one looks, this radiation comes from non-uniformly moving charges. A simple case to look at would be a linearly oscillating dipole. Equally charged positive and negative charges linearly oscillating about each other will experience non-uniform movement. This produces electromagnetic radiation with curved field lines emanating from the positive charge to the negative charge.

![Figure 1: A linearly oscillating dipole.](image)

As the charges oscillate closer to each other, the field lines get closer to closing. Eventually the displacement of the charges from each other is zero. At this point, the field lines close in on themselves.

![Figure 2: A linearly oscillating dipole with zero displacement.](image)
All the while, the radiation continues to expand away from the point of origin. The energy is radiated most strongly perpendicular to the direction of acceleration.

An atom is perhaps the most fundamental process for radiation emission and absorption. Here a dipole moment is formed between the negative electron cloud and the positive nucleus. An atom can react to incoming energy in two ways, absorption and scattering. Absorption in atoms occurs only at discrete energies. An atom can only absorb energy that is exactly the difference between its ground state and another excited state. Upon reaching the excited state, the atom quickly descends into a lower ground state. In the process of doing this, energy is released as a photon. The energy of this photon is exactly equal to the energy decrease of the atom. By the equation

$$\Delta E = h\nu$$  \hspace{1cm} (1)

it can be seen that since only certain $\Delta E$ are allowed, the emitted light will only be at certain frequencies. These frequencies are often called resonance frequencies.

When the magnitude of the incident radiant energy is not the correct $\Delta E$ for absorption, scattering can occur. The interaction causes an oscillation of the atom. The frequency of oscillation will be at the frequency of the initial electromagnetic radiation. The atom is then considered to be an oscillating dipole which then radiates energy at the same frequency. An oscillating atom, in simplicity, can be considered a source of electromagnetic radiation.

A dielectric within an external electric field will polarize and produce its own internal field; the total electric field comprising of the net field. The actual polarization of the dielectric is

$$\mathbf{P} = (\varepsilon - \varepsilon_0)\mathbf{E}$$  \hspace{1cm} (2)

The symbols $\varepsilon_0$ and $\varepsilon$ are respectively the permittivity of free space and the permittivity of the dielectric.

Despite the polarization, the electron cloud is bound to the atom and will oscillate about its equilibrium position when disturbed. In order for that oscillation to occur, there must be some sort of a restoring force of the form $F = -k_E x$. Here $k_E$ is the equivalent of a spring constant. It will oscillate at a resonant frequency of

$$\omega_0 = \sqrt{\frac{k_E}{m_e}}$$  \hspace{1cm} (3)
where $m_e$ is its mass.

The electric field of electromagnetic radiation exerts a force on the electron

$$F_E = q_e E(t) = q_e E_0 \cos(\omega t) \quad (4)$$

We combine this equation with Newton’s Second Law to get

$$q_e E_0 \cos(\omega t) - m_e \omega_0^2 x = m_e \frac{d^2 x}{dt^2} \quad (5)$$

where $q_e E_0 \cos(\omega t)$ is the driving force and $m_e \omega_0^2 x$ is the restoring force.

A potential solution to this equation could be $x(t) = x_0 \cos(\omega t)$. Inputting this into the equation above will yield

$$x(t) = \frac{q_e/m_e}{\omega_0^2 - \omega^2} E(t) \quad (6)$$

where $E(t) = E_0 \cos(\omega t)$.

The electric polarization can also be written as $P = q_e x N$ where $N$ is the number of contributing electrons per unit volume. Substituting our solution for $x(t)$ above into our equation for electric polarization, produces

$$P = q_e^2 N \frac{E/m_e}{\omega_0^2 - \omega^2} \quad (7)$$

Relating this to our original equation for electric polarization will give

$$\epsilon = \epsilon_0 + \frac{P(t)}{E(t)} = \epsilon_0 + \frac{q_e^2 N}{m_e \omega_0^2 - \omega^2} \quad (8)$$

Knowing that $n^2 = \epsilon/\epsilon_0$, the dispersion equation will follow

$$n^2(\omega) = 1 + \frac{Nq_e^2}{\epsilon_0 m_e} \left( \frac{1}{\omega_0^2 - \omega^2} \right) \quad (9)$$

The form of the dispersion equation above is sufficient for an independent atom. If there is more than one atom, then the electromagnetic radiation scattered by the other atoms must be accounted for.
As such

\[
\frac{n^2 - 1}{n^2 + 2} = \frac{N q_e^2}{3 \varepsilon_0 m_e} \sum_j \frac{f_j}{\omega_0^2 - \omega^2 + i \gamma_j \omega} \tag{10}
\]

V.G. Veselago gives the dispersion equation as

\[
\left| \frac{\omega^2}{c^2} \varepsilon_{ij} \mu_{ij} - k^2 \delta_{ij} + k_i k_j \right| = 0 \tag{11}
\]

where \(\omega\) is the frequency of a monochromatic wave and \(k\) is its wave vector. From the dispersion equation, it can be seen that the index of refraction is a function of frequency, permittivity, and permeability.

Here we turn to David J. Griffiths [6]. Electric permittivity (dielectric constant) \(\varepsilon\) and the magnetic permeability \(\mu\) are fundamental properties of an electromagnetic wave and affect its propagation. The electric permittivity relates to the interaction of an electric field with a dielectric. It is a measure of the resistance of the formation of an electric field within the medium or how well the atoms in a material reradiate energy received. For a linear medium, it can be expressed in the equation

\[
\mathbf{D} = \varepsilon \mathbf{E} \tag{12}
\]

In this equation \(\mathbf{D}\) is the electric displacement due to polarization and \(\mathbf{E}\) is the total electric field.

While the atoms in the material interact with the electric field, the permittivity (along with permeability) looks at the medium as bulk material.

Similarly, the magnetic permeability \(\mu\) relates to the interaction of a magnetic field with a material. It is a measure of the ability of a material to create an internal magnetic field. For a linear medium, it can be expressed in the equation

\[
\mathbf{B} = \mu \mathbf{H} \tag{13}
\]

In short, the electric permittivity and magnetic permeability make up the electromagnetic response of a material to electromagnetic radiation. These are usually determined by the atomic structure of the material, seeing as the atomic structure is typically much smaller than the wavelength of electromagnetic radiation. A metamaterial allows for manipulation of the permittivity and the permeability. Metallic inclusions are often the way this is accomplished. These inclusions manufacture the permittivity and permeability to desired values. The dimensions and spacing of the
inclusions are significantly smaller than the wavelength of the electromagnetic wave. As such the wave interacts with the metamaterial as a bulk material according to the permittivity and permeability of that metamaterial [4].

To understand the propagation of an electromagnetic wave, Maxwell’s equations are shown (here expressed for a vacuum)

\[
\begin{align*}
\nabla \cdot \mathbf{E} &= 0 \\
\nabla \cdot \mathbf{B} &= 0 \\
\n\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\n\nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{align*}
\]

(14)

The symbol \(\mu_0\) is permeability of free space. By taking the curl of the curl of both \(\mathbf{E}\) and \(\mathbf{B}\), the following equations will be obtained

\[
\begin{align*}
\nabla^2 \mathbf{E} &= \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \\
\nabla^2 \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}
\end{align*}
\]

(15)

For the derivation of these equations, please see Griffiths [6]. From a comparison of equations (15) to the three-dimensional wave equation

\[
\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}
\]

(16)

it can be seen that

\[
v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}
\]

(17)

Using the known values for \(\epsilon_0\) (8.85 \(\times\) 10\(^{-12}\) \(s\)\(^2\) \(\cdot\) \(C^2/m^3\) \cdot \(kg\)) and \(\mu_0\) (4\(\pi\) \(\times\) 10\(^{-7}\) \(m\) \cdot \(kg/C^2\)), it then follows that \(v \approx 3 \times 10^8 \text{ m/s}\) or

\[
\frac{1}{\sqrt{\epsilon_0 \mu_0}} = c
\]

(18)

Thus, in free space the electromagnetic wave will be traveling at the speed of light.
For electromagnetic waves propagating through matter, Maxwell’s equations are become

\[ \nabla \cdot \mathbf{D} = 0 \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{H} = \mu \epsilon \frac{\partial \mathbf{D}}{\partial t} \] (19)

No longer being in free space, \( \mu_0 \) and \( \epsilon_0 \) have been replaced by the permeability \( \mu \) and the permittivity \( \epsilon \) of the material.

At this point it becomes necessary to refer to Veselago [1]. For a monochromatic plane wave, the electric and magnetic fields can be represented respectively as

\[ \mathbf{E} = \mathbf{E}_0 e^{i(kz - \omega t)} \]
\[ \mathbf{B} = \mathbf{B}_0 e^{i(kz - \omega t)} \] (20)

Please note the both are proportional to \( e^{i(kz - \omega t)} \). Equations (20) can be combined with equations (12) and (13) and inserted into the last two equations in (19) resulting with

\[ \mathbf{k} \times \mathbf{E} = \frac{\omega}{c} \mu \mathbf{H} \]
\[ \mathbf{k} \times \mathbf{H} = -\frac{\omega}{c} \epsilon \mathbf{E} \] (21)

Veselago here points out that when \( \epsilon > 0 \) and \( \mu > 0 \), the \( \mathbf{E}, \mathbf{H}, \) and \( \mathbf{k} \) vectors form a set that could be visualized using your right hand to make three vectors and characterizes this as “right-handedness”.

He then states that if \( \epsilon < 0 \) and \( \mu < 0 \), those vectors form a set that could be visualized by using your left hand to make three vectors. He characterizes this as “left-handedness” [1]. Wave propagation through a material can be described as a matrix where \( \alpha_i, \beta_i, \) and \( \gamma_i \) represent the direction cosines for the vectors \( \mathbf{E}, \mathbf{H}, \) and \( \mathbf{k} \) respectively

\[ \mathbf{G} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \] (22)

Denoting the variable \( p \) to represent the “handedness” or “rightness” of a material, for right-handed materials \( p = +1 \) and for left-handed materials \( p = -1 \). These values are equal to the determinant of the \( \mathbf{G} \) matrix [1].
The Poynting Vector ($\mathbf{S}$) tells us about the energy that is being transported by the electromagnetic wave. Its direction is determined by the directions $\mathbf{E}$ and $\mathbf{H}$ as shown in the relationship

$$ \mathbf{S} = \mathbf{E} \times \mathbf{H} $$ (23)

From Equation (23) it can be seen that $\mathbf{S}$ is perpendicular with $\mathbf{E}$ and $\mathbf{H}$, being right-handed. Thus, for right-handed materials, $\mathbf{k}$ and $\mathbf{S}$ are in the same direction. For a left-handed material, $\mathbf{k}$ and $\mathbf{S}$ would then be in opposite directions. Knowing that the $\mathbf{k}$ vector is in the direction of the phase velocity, it holds that a left-handed material could be defined as one in which the group velocity is negative.

1.2.2 Refraction

When an electromagnetic wave propagates from one medium to another, there are boundary conditions that must be satisfied. Namely

$$ \mathbf{E}_i = \mathbf{E}_t $$

$$ \mathbf{B}_i = \mathbf{B}_t $$

$$ \epsilon_i \mathbf{E}_i = \epsilon_t \mathbf{E}_t $$

$$ \mu_i \mathbf{B}_i = \mu_t \mathbf{B}_t $$ (24)

From these equations it can be seen that the x-direction and y-direction cannot vary from the initial wave to the transferred wave at the boundary. This is independent of the rightness of the materials. However, the variance of the z-direction will be dependent upon the rightness of the two materials in question. If the materials have opposite handedness (one is right-handed and the other left-handed), then the z-direction will change sign. This change will occur in the $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{k}$ vectors. Veselago says that “this corresponds to the fact that in passage into a medium of different rightness the vectors $\mathbf{E}$ and $\mathbf{H}$ not only change in magnitude owing to the difference in $\epsilon$ and $\mu$ but also undergo a reflection relative to the interface of the two media” [1]. This can be seen in the following figures
1.2.3 Snell’s Law

The Law of Refraction or Snell’s Law relates the incident and transferred angles of an electromagnetic wave (relative to the normal of the surface of the transferring edge) to the index of refraction of both the incident and transferring mediums.

\[ n_i \sin \theta_i = n_t \sin \theta_t \]  \hspace{1cm} (25)

or

\[ \frac{n_{ti}}{n_i} = \frac{n_t}{\sqrt{\varepsilon_i \mu_i}} = \frac{\sin \theta_i}{\sin \theta_t} \]  \hspace{1cm} (26)

In the form above, Snell’s law does not account for the rightness of the mediums and as such cannot accurately depict what happens when an electromagnetic wave propagates through two medium of opposing rightness. Veselago states the correct form to be

\[ n_{ti} = \frac{p_t}{p_i} \frac{\sqrt{\varepsilon_t \mu_t}}{\sqrt{\varepsilon_i \mu_i}} = \frac{\sin \theta_i}{\sin \theta_t} \]  \hspace{1cm} (27)

where \( p \) represents the rightness of each medium as previously discussed [1].
1.3 Significance and Applications of Metamaterials

There has been a recent increase of interest in metamaterials. This is due to a variety of significant applications that can result from metamaterials. As previously mentioned, these include the ability to tailor the electromagnetic properties of the material, metamaterials used as a perfect lens, and cloaking.

Metamaterials are typically constructed by using some base material (typically PCBs) that has metallic inclusions. These metallic inclusions vary in size and shape but are a repeated pattern throughout the material. The size and shape of these inclusions are what determines the electromagnetic properties of the metamaterial [1]. This allows for tailor-made materials that have the specific desired properties, even if such a material is not typically found in nature.

In optical imaging, higher resolution is obviously better. However, there is a limit that most optical imaging systems hit that is dependent upon the wavelength of light. The diffraction that results causes imaging resolution of any system to have what is called the diffraction-limit. Current research is looking into the possibility of metamaterials being able to achieve resolution beyond this diffraction-limit.

In order to cloak or make an object invisible, an electromagnetic wavefront that hits the object will have to bend around the object and then reconstruct the original wavefront. This presents a problem.

Figure 5: A wavefront bending around a circular object. Progression is from left to right.

In Figure 5, the wavefront progresses from left to right. The part of the wavefront in the immediate path of the circular object will have to travel faster to go around the object than the part of the wavefront that is not in the path of the object. If not, the wavefront will not be reconstructed properly and you will be able to detect the object.
Since the part of the wavefront not interacting with the object is already traveling at the speed of light, it holds that the part of the wavefront that does interact with the object must travel faster than the speed of light. Metamaterials can be so constructed to yield $|\mu| < 1$ and allow for this.

Metamaterials are able to provide better control of the propagation of light. By so doing, you would be able to direct light to bend completely around a given object and continue in its original path as if the object wasn’t even there. One of the current limitations with this is that constructed metamaterials operate optimally at a certain resonant frequency. Since light can have a very wide variety of frequencies, this is a problem. Your metamaterial might work well with red light at 700 nm but it might have a difficult time with light at 650 nm let alone something of a different color like green at 525 nm. Current research is looking into this.
Chapter 2

Design of Materials Used

2.1 Microwave Optics Apparatus

For this project, there are several materials that were needed. Critical was some sort of a microwave optics apparatus. The BYU-Idaho Physics Department has such equipment.

A TX2 (transmitter) and a RX2AS (receiver) Microwave Apparatus by IEC was used. The transmitter is rated at 10.7 GHz and was experimentally shown to be 12.5 ± 5 GHz. It needs to be powered by a 12V power source. The receiver has a 9V battery to power it. Using an external connection, a sine wave signal generator was connected to the transmitter. A DAQ was connected to the receiver by two alligator clip cables. The DAQ is connected to a computer and the voltage is recorded in LABVIEW and exported to an Excel spreadsheet. For more details on the use and operation of the transmitter and receiver, please see Allen Andersen’s BYU-Idaho senior thesis, Appendix B, “Microwave Optics Equipment Manuel” [7].
In addition to the already listed equipment, two RF foam apertures were used. The foam has a pyramidal shape (reduces creation of standing waves). Each foam aperture has two 60.96m $\times$ 35.56m solid sheets attached to pegboards covered in aluminum foil. In the middle of each aperture is a circular hole with a 5cm diameter [7]. Two wooden blocks were used to elevate both the transmitter and the receiver each.

Also used were a goniometer base and a compass sheet. The goniometer base is made of three wooden boards with slits connected by swivels. The bottom board has a slit cut into it that allows for a meter stick to be placed in it. The middle board also has a slit cut into it that allows for meter stick placement and can turn independently of the top and bottom board [7]. The goniometer base rests on top of the compass sheet. The two RF foam apertures surround the goniometer.
All of this in combination allows for: the stationary placement of the transmitter on one side of a RF foam aperture, the goniometer placement in between the two foam apertures, and the remaining RF foam aperture and the receiver to be placed and rotated at different angles (with respect to the goniometer) without having to change the rest of the equipment. My metamaterial is placed on the goniometer, in between the two RF foam apertures.

Figure 9: Setup of RF foam apertures, compass sheet, and goniometer.

Figure 10: Setup of all equipment (minus metamaterials)
2.2 Metamaterial Design

Over the past decade or two, metamaterials have become a more common area of research. A variety of different designs have been used ranging from square meshes [8] to split-ring resonators (SRR) [9] to even random patterns [10]. There was one particular design that caught my interest. This original design was published by Christian Imhof and Remigius Zengerle. It is a double crossbar inclusion on both sides of a printed circuit board (PCB) [11]. The designs I used were largely built on this.

My metamaterial design is also composed of a PCB with a repeated inclusion of copper throughout the board. Each copper inclusion will be considered a unit cell. Each of the bars in the crossbar is 7mm long and 1.2mm wide. Each unit cell is 7.30mm×7.30mm [4].

![Figure 11: Unit cell of metamaterial.](image)

The output wavelength of the transmitter was rated at 2.8cm and experimentally shown to be 2.4 ± 0.4cm [7]. Thus, the metallic inclusions are much smaller than the wavelength of the microwave beam and the beam will treat the metamaterial as a bulk medium.

With multiple boards, the unit cell has depth to it as well. The actual PCB board is 59 mil or 1.4986mm thick with the copper layers each being 1oz/ft² or 35μm thick. The unit cell is 4.4958mm thick. Thus, there will be a 2.9972mm space separating each board. Since each board does not stand well on its side on its own, each board will be surrounded by a PCB board (of the same type), stripped of its copper layers and cut to the same dimensions.
In order to better test this, a wedge of this material was created. This wedge was composed of many boards of various sizes.

The picture below is the final metamaterial. The base (and largest) board has a blank board on top to separate it from the board next to it but no blank board on the bottom of it. This gives the wedge a thickness of \( (88.0 \pm 0.1) \text{ mm} \). Its length is \( (149.0 \pm 0.1) \text{ mm} \). This gives the wedge an angle of about \( 32^\circ \). A rubberband was used to help the wedge keep its form when standing on edge but does not sufficiently hold it together when handling the wedge.
Due to the availability of materials, this wedge does differ from the designs given in Imhof and Zengerle’s paper. As stated above, the thickness of each individual board is 1.4986mm, giving a unit cell thickness of 4.4958mm. The original design has an individual thickness of 0.8mm, giving a unit cell of 3.87mm [4].

2.3 Metamaterial Construction

Once a specific design was determined for the metamaterial, construction could begin. To actually construct the metamaterial, the fabrication facilities of BYU-Idaho’s Electrical Engineering (EE) Department were utilized. They have a LPKF Protomat S103 which is a “circuit board plotter for producing PCB prototypes and... is configured specifically for RF and microwave requirements” [12]. They also had the programs necessary to design and format the metamaterial for the milling machine.

To get the designs made and formatted into a Gerber file, I worked with one of the lab assistants, Joseph Olivero Ramierz, for the EE Department. He introduced me to the program Altium. Using this program, twenty boards were designed of various sizes. The largest board was 20 unit cells × 11 unit cells and the smallest was 1 unit cell × 11 unit cells.
These designs were formatted as a gerber file. The lab assistant then had the machine mill them all out. FR4-59 mil boards with 1oz/ft² of doubled sided copper were used. While the machine did a good job, there were several pieces of copper that the machine did not remove. As much as it was possible, the extra copper was removed with a razor blade.

**Figure 15:** Table of board dimensions

<table>
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<tr>
<th></th>
<th></th>
<th></th>
<th>In unit cells</th>
<th>In millimeters</th>
<th>In inches</th>
<th>in²</th>
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</tr>
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</table>

**Figure 16:** Look to the right of the board shown, an extra strip of copper can be seen. Such impurities existed throughout each board and had to be carefully removed by hand as best they could.
The blank PCB boards were obtained in a different manner. Several sheets of the same FR4 board that was used print the metamaterial boards were ordered. These sheets also had the double sided copper. While FR4 boards with less copper could have been ordered, they were more expensive than the double sided copper boards. Milling is not the only method of removing copper from a PCB. This can also be done through the process of chemical etching.

In chemically etching a PCB, one places a cover over the copper design that is desired to remain on the PCB while entire board is immersed or scrubbed with chemicals that remove the remaining copper from the board. One such chemical is FeCl$_3$ or ferric chloride. This chemical is available from Radio Shack at a fairly inexpensive price.

Since blank (no copper whatsoever) boards were desired, no cover was used to preserve copper patterns.

The reaction taking place is a two-step reduction/oxidation reaction. First the ferric chloride reacts with the copper to form copper (I) chloride

$$ \text{FeCl}_3 + \text{Cu} \rightarrow \text{FeCl}_2 + \text{CuCl} $$

Then the ferric chloride reacts with the copper(I) chloride to form copper (II) chloride

$$ \text{FeCl}_3 + \text{CuCl} \rightarrow \text{FeCl}_2 + \text{CuCl}_2 $$

Upon receiving the boards, BYU-Idaho EE Department facilities were used again to cut the boards to have the appropriate dimensions. The dimensions of each board cut out matched the dimensions listed in the above table. In term of unit cells, two $1 \times 11$ boards were cut, two $2 \times 11$ boards were cut, two $3 \times 11$ boards were cut, all the way to the $20 \times 11$ boards. After the boards were cut, Mike Wood from the BYU-Idaho Chemistry Department allowed me access to the department’s facilities. A fume hood was provided along with lab goggles, a lab coat, gloves, wash bottles and other materials. Ferric chloride was then used to strip the copper off of each of the boards. While a chemical bath alone could have been used to strip the copper, I used a slightly different process to speed up the reaction.
Several boards were placed in a plastic container. Ferric chloride was then poured on top of the boards until they were covered. Using the soft side of a sponge, I would then rub the copper on one side while the other boards sat in the bath. This allowed for the bath to begin the chemical reaction on some of the boards while I rubbed another board to speed up the reaction for that board. The more boards that were stripped, the more saturated the solution became with the precipitant. As such it was less effective as time progressed and the reaction took longer. At this point, a wash bottle was used to rinse the excess chemicals off of the completed boards and then the used chemicals were stored by the Chemistry Department to be disposed of later.

Figure 17: Chemical bath for boards with copper on them being prepared. The boards were also rubbed with a sponge to quicken the chemical reaction taking place.
Chapter 3

Methods and Results

3.1 Equipment Setup

All of the equipment described in the previous chapter was used. It was set up as there described. In this setup, a laptop was connected to DAQ. This was done in light of the need to take several measurements over a variety of angles. Thus necessitating the need to move the DAQ and the computer it was connected to several times. When positioning the laptop, per instructions from Allen Anderson’s senior thesis, do not set up the laptop in line with the beam, even though there are foam apertures and a receiver between the beam and the laptop [7].

To make sure things didn’t move during the process of collecting the data, the base of the goniometer was taped to the angle wheel. A meter stick was inserted into the bottom slit on the goniometer to serve as a guide for the transmitter. The goniometer was taped down such that the ruler ran along 0°. The meter stick was pushed in until the 10cm mark was visible in the center hole of the goniometer. Another meter stick was inserted into the other slit in the goniometer and was pushed in until the 10cm mark was visible in the center hole. It could be positioned at any desired starting angle.
One of the foam apertures was placed along the bottom meter stick as close as possible to the goniometer, helping to minimize the rotation of the goniometer during the experiment. The transmitter was then placed to be 5cm from the foam aperture. The wavefunction generator was set to a square wave of \((100.06 \pm 0.02)\text{Hz}\) and connected to the transmitter. The second foam aperture was placed at approximately 40.0 cm and the receiver was placed to be 5cm from the foam aperture. The receiver was initially placed on GAIN 1 but it was decided that GAIN 2 would give a better reading.

A problem could be seen from initial setup of the equipment. The center of the actual aperture in the foam was 15.4cm higher than the top of the goniometer. The metamaterial wedge is only 8.30cm tall. As such, the metamaterial needed to be elevated so that it would be directly in between the two apertures. To accomplish this, two small Styrofoam blocks were positioned such that smaller materials could be placed on top of them, elevating them to be directly in between the two apertures (directly in the path of the beam). This added 9.90cm of height, allowing for better placement of the metamaterial wedge.

3.2 Code

All code can be found in Appendix A – Code. In order to collect data, there are two LabVIEW files of code that are needed. One is the actual data collection program. The other file is the program used to save the data as an Excel “.csv” program. The Program 1 in Appendix A is the data collection program.

Program 2 in Appendix A sets the file properties of the exported data. While it needs to be in the same file location as the first file to actually save the data as an Excel “.csv” file, it does not need to be open and running it order for the data collection program to work. This code is built such that each time data is collected (each angle) a new Excel “.csv” file was created. Before the data can be analyzed, it needs to be compiled to one Excel “.xlsx” file and modified so that each column has the same number of rows. So after all the data was in one file, the angle (column) with the least amount of rows was found and all subsequent data (rows) following were deleted. LabVIEW code was originally obtained and used by Ross Blaszczyk for his BYU-Idaho senior research project.
After the data was collected and prepared, a MATLAB program was used to import the data and analyze it. This is Program 3 in Appendix A. After importing, the code averages the data for each angle. It produces two plots. The first plot is of the voltage at each of the actual measured angles. The second plot is of the voltage in terms of the refracted angles.

3.3 Process

The process used to collect data was as follows. After everything was hooked up, LabVIEW code was used to collect the actual data. The LabVIEW code collected data at the rate of 1000Hz. I would run the code for 5-6 seconds and then move the rotatable meter stick, the foam aperture and the receiver all at once to the next angle. The LabVIEW code would export and save the data collected to an Excel file as a “.csv”. However, the code I used would create a new Excel file of collected data for each angle. Thus, I recommend when it prompts you to save the file, you include in the name of the file the actual angle you are collecting data for.

Since a new Excel file is created for each angle, it was decided that preliminary measurements would be taken with degree variance of approximately 10° and then 5° to ensure the equipment was set up correctly, the code is running properly and that the beam is behaving in an “expected” manner. After the preliminary measurements, degree variance was reduced to approximately 1° for greater accuracy.

If while running the LABVIEW code, you come across “Error _ 200361”, please see Appendix B.

Since the space between the two foam apertures is no longer “empty”, it was decided to take some initial readings with nothing but the Styrofoam blocks on the goniometer to determine if and/or how much they were refract or reflect the beam. Preliminary readings were taken over a spread of 120° or ±60° off of 180° (predicted beam path with no medium interference).

3.4 Results

In collecting the data, each angle was measured with uncertainty of ±0.5°. Since the voltage is proportional to the intensity, the data was left in terms of voltage. Uncertainty is included in each figure description.

The first graph shown contains preliminary data. Data was taken when there was nothing on the goniometer. Starting at 115°, measurements were taken every 5° until 245°. It can be seen that the
peak of the beam is at 185°. This data gives us an initial standard to go against, taking into account any background radiation that might be affecting the beam.

The second graph shows the measurements taken when the Styrofoam blocks were placed on the goniometer. They were place flush with the RF foam aperture closest to the transmitter and centered accordingly. Measurements were taken starting at 120° until 225°. Data was taken every (1.0 ± 0.5)°. It can be seen that the beam is most intense between 170° and 200°.
Subsequent testing from 160° to 200° was done, collecting data every \((1.0 \pm 0.5)^\circ\). The measurements taken are shown in the following graph.

From these measurements it can be seen that the beam is most intense at 184°. Thus we can see that the Styrofoam blocks causes little to no variance in the path of the beam. Subsequent refraction of the beam seen with the metamaterial wedge on the blocks is resultant of the metamaterial itself.

The process was repeated again, this time with the metamaterial wedge on the Styrofoam blocks. The wedge was placed to be flush with RF foam aperture. As such its position was as perpendicular with the incident microwave beam as possible. This limited the refraction of the beam to occur only as the beam leaves the metamaterial wedge. Data was collected over a 110° spread, ranging from 90° to 200°, every \((1.0 \pm 0.5)^\circ\).
The graph below is in terms of refracted angles.

The process was repeated again. This time the wedge was positioned such that the angled edge was over the center of the goniometer and compass wheel. Data was collected over a spread of 100°, ranging from 100° to 200°, every (1.0 ± 0.5)°. Initial data is shown in the following graph.
This allowed for better accuracy in using the goniometer and compass wheel to determine the angles at which the measurements were taken, relative to where the beam initially refracts. To determine the degree at which the normal of the wedge was at, the following process was used. A path from the center of the actual aperture in the foam was followed to see where exactly on the wedge the beam would leave. Then, a protractor was used to find the normal of the wedge. The normal line coincided with $143 \pm 1^\circ$. This was the angle used to determine how much the beam had refracted. The refracted angles are shown in the graph below.

Figure 23: Graph of microwave beam with metamaterial wedge on Styrofoam blocks. Here the wedge was placed such that where the beam would refract out of the material was centered to be over the goniometer. Measurements were taken every 1 degree.

Uncertainty in Voltage: $(0.2-3.2) \times 10^{-4}$
The highest peak occurs at -4°. There are 4 other peaks of lesser value but relatively close to each other. They occur at -11°, 8°, 18° and 38°/39°. At this point in the project, the microwave receiver stopped working and was not able to accurately collect any more data. It is currently being looked at for repairs.
Chapter 4

Conclusion

4.1 Explanation of Results

By comparing Figures 21 and 22 with Figures 23 and 24, it can be seen that there is a discrepancy in the shape of the data. The latter two figures contain multiple peaks while Figures 21 and 22 do not. An explanation of this discrepancy could be the fact that in Figures 21 and 22, the wedge was positioned close to be flush with the RF foam aperture. Hence, the microwave beam that went through the circular aperture did not spread out but immediately entered the metamaterial. In Figures 23 and 24, the wedge was placed to be centered on the goniometer. Thus the microwave beam that went through the aperture was able to spread and could have possibly caused interference with the actual refracted beam.

Without being able to take any additional measurements, the data that has been collected is insufficient to verify that the material is in fact a metamaterial. It is possible that the material might not be a metamaterial. There are several explanations why this could be the case. The most probable explanation is due to the difference in the actual designs of the boards.

One of the limitations of metamaterials in general is that they respond to a certain resonant frequency. Thus, if a metamaterial is resonates with blue light, it is only for that particular shade of
blue. Different shades would cause the metamaterial to be less effective and different colors might cause the metamaterial not to work at all.

In the paper the designs were taken from, the board had a thickness of 0.8mm. Prior to testing the design, a commercial solver for Maxwell’s equations (CST Microwave Studio) was used [4]. From this, it was predicted that the metamaterial design resonates at 10.1 GHz and experimentally shown to be 10.2 GHz. The difference is the size of the boards could have affected the frequency that it is optimized for. Thus, while I am testing for approximately 10 GHz, it could be a significantly differently frequency at which the material is a functional metamaterial.

4.2 Limitations

Here I address the limitations faced throughout the research project. The first limitation was the thickness of the PCBs that were used. As described in Section 2.2, the thickness of my PCB board is 1.4986mm while the thickness of the PCB in the paper is 0.8mm. My use of a thicker board was due to the available material from the BYU-Idaho Electrical Engineering Department. This limitation most likely had the biggest impact on the results.

The use of such thin boards also added difficulty to the experiment. Each board needed to be spaced on the order of $10^{-3}$ m. As a solution to this problem, it was decided to use PCB boards of the same type (FR4) cut out to the same dimensions as the metamaterial PCBs and stripped of all copper as spacers. While makes it easier for the boards to stand up on its own, a rubberband was still necessary to ensure the boards stood up on.

The machines used to construct material did not perfectly remove all of the unnecessary copper. As mentioned earlier, I found several boards that had copper remaining on it and sought to remove the extra copper with a razor blade. The chemical bath that was used possible could have left a residue of copper that I was unable to see. The potential of impurities within the material itself likely affected the results.

4.3 Future Research

Future research could take several different avenues. This could include addressing the limitations. First and foremost, fixing the microwave receiver would allow for additional testing to be done to verify the initial results that have been collected.
This experiment could also be repeated but using the correct thickness. Doing this would allow the experiment to be conducted while exactly matching the designs described in the paper. This should confirm the results obtain therein.

Another potential area of research would be determining the frequency at which the metamaterial built resonates. Solving Maxwell’s equations would tell you this and knowing this, the experiment could be redone at the appropriate frequency.

Other designs could be made and tested for a variety of frequencies. It could even be possible to replicate past experiments done to encase an object with metamaterials to verify cloaking. Original designs could also be developed and tested.

While not enough data has been collected to verify that this material is a metamaterial, the data that has been collected certainly shows promise. Further testing with working equipment could verify the metamaterial to have a negative index of refraction. These results are still exciting as they show that it is possible for students in BYU-Idaho’s Physics Department to design, construct and test metamaterials at the microwave spectrum, something that is a current area of prominent research.
Appendix A

Code

PROGRAM 1

Front Panel for the data collection LABVIEW program
Block Diagram for the data collection LABVIEW program.
PROGRAM 2

Front Panel for the file specifications LABVIEW program

[Image of Front Panel]

Block Diagram for the file specifications LABVIEW program.

[Image of Block Diagram]
PROGRAM 3

This is the MatLAB program to import collected data and analyze it.

```matlab
%*****************************************************
% Jordan Claspell
% Faculty Advisor: Brother Todd Lines
% Metamaterial Research
% Data Import and Fast Fourier Transform (FFT)
% The purpose of this program is to import the data obtained by LabView &
% stored in Microsoft Excel and to then interpolate this data using a FFT.
%**************************************************************************

%% Main Function
clear; clc;

fprintf('Input filename (start and end name with single tic-marks)\n');
filename = input(' (Example: 'BlankTest1.xlsx')\n-- ');

sheet = 1; %what sheet in Excel you are referencing
DATA = xlsread(filename,sheet); %import data from Excel into MATLAB

angles = DATA(1,:).'; %copy angles into their own array
DATA(1,:) = []; %
a = size(DATA); %find size of array to use in averaging the data

avgDATA = (sum(DATA)/a(1)).'; %find the average of the data for each angle
refAngle = angles - 143; %find refracted angle

figure;
plot(angles,avgDATA,'-d')
set(gca,'XTick', (angles(1) - 5):10:(angles(end)+5));
title(['Measured Angles of ''', filename, '''']);
xlabel('Angle');
ylabel('Voltage');

figure;
plot(refAngle,avgDATA,'-d')
title(['Refracted Angles of ''', filename, '''']);
xlabel('Angle of Refraction');
```

54
ylabel('Voltage');

To determine the uncertainty for each data set that was collected, the following code was added.

```matlab
x = 0; % initial x-value for sum
for (j=1:a(2))
    for (i=1:a(1))
        diff = DATA(i,j) - avgDATA(j); % take current value and subtract from average value
        diff2 = diff^2; % square difference
        x = x + diff2; % take sum of all the squared differences
        s = sqrt(x/(a(1)-1)); % find standard deviation
        S(j) = s/sqrt(a(1)); % find standard deviation of the mean
    end
end
figure;
plot(angles,S)
```
Appendix B

Programming Error and Resolution

Throughout the process of collecting data, I occasionally got this error:

“Error __ 200361

Possible reason(s);
Onboard device memory overflow. Because of system and/or bus-bandwidth limitations, the driver could not read data from the device fast enough to keep up with the device throughout.

Reduce the sample rate, or reduce the number of programs your computer is executing concurrently.

Task Name: _unnamed Task <5>”

There are two potential solutions to this error. The first is to go to the “Tools” menu and to click on “Measurement & Automation Explorer”. This will bring up a new window entitled “My System – Measurement & Automation Explorer”. On the very left side of this window, under “My System”, click the arrow to the left of “Devices and Interface”. At the bottom of the list it brings up, it should say “Network Devices”. Above that, there should be a list of all the DAQ devices that have been used on that computer. Each one will appear as “NI USB-6008 “Dev1”” or “Dev 2” or “Dev3”, etc. If there is a red “x” next to the name, then click on the name and press the “Delete” key. Only keep the active device in use. Close the “My System – Measurement & Automation Explorer” window and the program should now run.

Even after doing this, the program would sometimes still give me this error. I initially would unplug the DAQ, go through the solution process described above, plug the DAQ back in and under “Physical Channel” in the actual program reselect the correct DAQ output channel. Eventually, I discovered that simply unplugging and plugging the DAQ back in would work as well. The first time this error appears, I would recommend the first process to clear any unused devices and then use the second solution for subsequent errors.
Appendix C

Uncertainty

To determine the uncertainty in each data set that was collected, it was decided to find the standard deviation of the mean. The process that I used for data collection was thus. Data was collected for approximately 5-6 seconds at each angle at a rate of 1000Hz. The average data-value was found for each angle. My graphs of the data do not represent the data actually collected but rather represent the average data collected at each angle. Hence, it is more appropriate to calculate uncertainty by using the standard deviation of the mean as opposed to simply the standard deviation.

To compute the standard deviation of the mean, the following equations were used:

\[
\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}
\]

This equation allows us to find the standard deviation ($\sigma$). $x_i$ represents the actual data-value in the data set. $\bar{x}$ represents the mean value for the data set. $n$ represents the total number of values in the data set.

\[
\frac{\sigma}{\sqrt{n}}
\]

This equation gives us the standard deviation of the mean.

The following graphs are the standard deviation of the mean for each of the graphs. The x-axis for each graph is the angles at which data was collected. They y-axis is the standard deviation of the mean at each angle (in volts).
For Figure 18:

For Figure 19:
For Figure 20:

For Figures 21-22:
For Figures 23-24:
References


