Fall 2017

Math Boot Camp
Functions and Algebra

FUNCTIONS

Much of mathematics relies on *functions*, the pairing (relation) of one object (typically a real number) with another object (typically a real number).

Commonly functions are visualized as producing an output (usually outputs are represented with $y, f(x), g(x),$ etc.) when given an input (often x is used to represent the inputs)

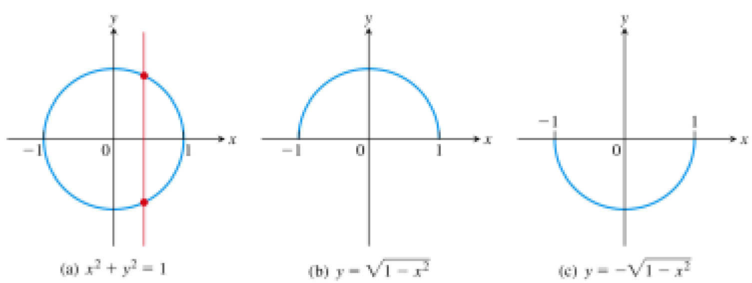
All numbers (or elements) that can be used as inputs form a set of numbers called the *domain*. The outputs are called the *range*.

Definition: A **function** f from a set D (the domain) to a set Y (the range) is a rule that assigns a $unique$ (single) element y in Y to each element x in D.

Examples:

1) $y = 2x$ is a function because there is exactly one output, y, for every input, x.

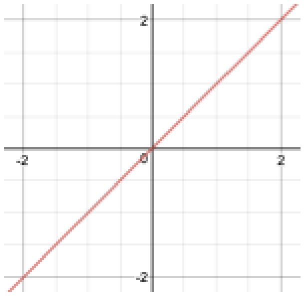
2)



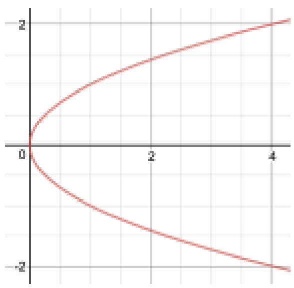
Note: The circle (a) is <u>not</u> a function as each x-values is assigned to two y-values (as indicated by the red vertical line). The semicircles (b) and (c) are both functions.

Practice: Which of the following are functions?

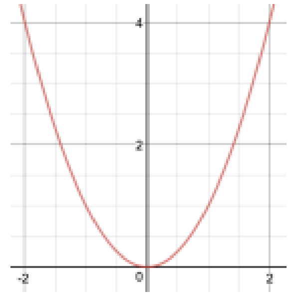
a)



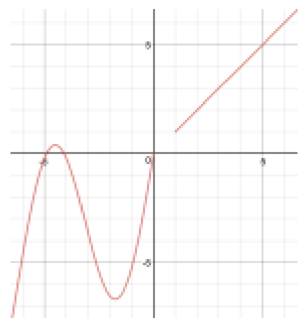
b)



c)

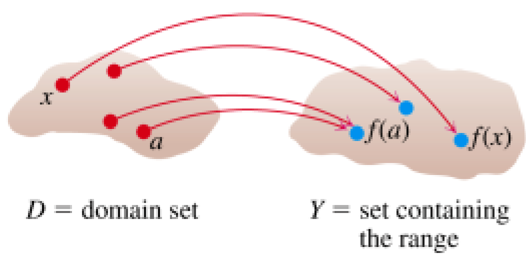


d)



DOMAIN AND RANGE

Recall that a function assigns each number in the domain (input) to exactly one number in the range (output).



However, not every number can be used as an input. For example, zero cannot be used as an input if a function assigns the output, y, to one divided by the input, x, or written mathematically

$$y = \frac{1}{x}$$

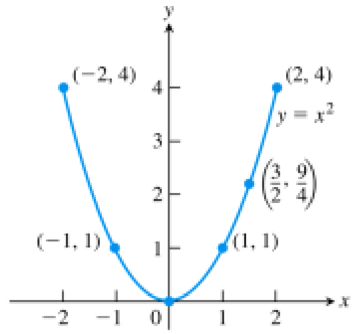
(we cannot divide by zero). Thus, zero is not in the domain.

Furthermore, a function's range is often limited as well. For example, while

$$y = x^2$$

has an unrestricted domain (all real numbers, denoted $(-\infty, \infty)$ or $\mathbb{R}$) but the range is non-negative numbers (denoted $[0, \infty)$).

x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4



Additional examples of domains and ranges of common functions are listed below.

Function	**Domain (x)**	**Range (y)**
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

Practice: What is the domain and range of the following functions?

a) $y = x$

b) $y = x^3 + 3x^2 - 2$

c) $y = \sqrt{4x}$

d) $y = |x|$

e) $y = \frac{1}{(x-9)^2}$

Common Functions:

These functions described in the applications below are very common and you will be expected to know well in calculus. In the applications, the functions are represented by name, with data, on a graph, with a story and in an equation. Explore these common functions using the following apps

Function Machine App
https://content.byui.edu/file/475bf42b-5122-4ac6-97fa-9dec9ab8cbf0/22/function-machine.zip/function-machine/index.html?file=funcMachineSettings&load=general

Function Family App
https://content.byui.edu/file/9324143a-041f-40e4-97f5-8a3f23afa880/1/index.html?f=pieper

Function Notation:

Most students are familiar with functions written as $y = 3x + 2$. However, some feel much more uncomfortable when the same function is written $f(x) = 3x + 2$. For functions, the two notations (y and $f(x)$) mean the exact same thing, but $f(x)$ gives you more flexibility and more information.

We can either write out:

Given $y = 3x + 2$; solve for y when $x = -1$ or

Given $f(x) = 3x + 2$; find $f(-1)$.

In either case you "plug" in -1 for x, multiply by 3 and add 2 to get the final value -1. To express the solution you can either write

$y = -1$ when $x = -1$, or

$f(-1) = -1$.

In function notation, the "x" in "$f(x)$" is called "the argument of the function", or just "the argument".

Examples:

Given $f(x) = 3x^2 + 2x + 1$, calculate the following

a) $f(2)$

Solution:
$f(2) = 3(2)^2 + 2(2) + 1 = 17$

c) $f(@)$

Solution:
$f(@) = 3(@)^2 + 2(@) + 1$

b) $f(-1)$

Solution:
$f(-1) = 3(-1)^2 + 2(-1) = 2$

d) $f(x+h)$

Solution:
$f(x+h) = 3(x+h)^2 + 2(x+h) + 1$

Multiplying Polynomials:

Distributive Property: An algebraic property which is used to multiply a single term by two or more terms inside a set of parenthesis.

Examples:

a) $2(3+6) = 2(9) = 18$

or

$2(3+6) = 2(3) + 2(6)$
$= 6 + 12 = 18$

b) $2(x+3) = 2(x) + 2(3)$
$= 2x + 6$

Multiplying Polynomials is done in two steps.

1) Distribute each term of the first polynomial to every term in the second polynomial. Remember your rules of exponents!
2) Combine any like terms.

Examples:

a) $3x^2(2x^2 + 4x + 1)$
$= 3x^2(2x^2) + 3x^2(4x) + 3x^2(1)$
$= 6x^4 + 12x^3 + 3x^2$

b) $(3x+1)(4x-2)$
$= 3x(4x) - 3x(2) + 1(4x) - 2(2)$
$= 12x^2 - 6x + 4x - 4$
$= 12x^2 - 2x - 4$

Factoring Polynomials:

Factoring polynomials involves breaking up a polynomial into simpler terms (undoing multiplication). When these terms are multiplied together they equal the original polynomial.

For example

a. $x^2 - 16 = (x-4)(x+4)$
Note: This is an example of "difference of squares." Polynomials of this type always factor like $x^2 - a^2 = (x-a)(x+a)$ for any a.

b. $x^2 + 2x + 1 = (x+1)(x+1) = (x+1)^2$

c. $4x^2 + 5x + 1 = (4x+1)(x+1)$

Fall 2017

Common Misunderstandings:

A. Determine if the following equations are true or false. If it is false, change the right side of the equation so that the equation is true.

1. $7(x+3) = 7x+21$
2. $(x-8)(x+10) = x^2 - 80$
3. $(2x+4)^2 = 4x^2 + 16$
4. $\sqrt{x^2+2x+1} = x + \sqrt{2x} + 1$
5. $(x-1)^3 = x^3 - 1$

B. Completely factor the following polynomials.

1. $27x^2 - 81x$
2. $x^2 - 3x - xy + 3y$
3. $12x^3 - 60x^2 + 75x$
4. $(x-3)^3(x+1)^5 - (x-3)^2(x+1)^6$

C. Completely simplify the following rational expressions.

1. $\dfrac{24x^2 + 18x^3}{12x^2}$
2. $\dfrac{18x^2y^3 - 42x^3y^2}{6xy^2}$
3. $\dfrac{1-2x}{2x^2+5x-3}$
4. $\dfrac{x^2-2x}{x^2+2x+1} \bullet \dfrac{x^2+4x+3}{x^2+3x}$