QUANTITATIVE REASONING:
OUR NEW MINIMUM MATHEMATICS REQUIREMENT

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I was invited to summarize the philosophy, curricular objectives, prospective audience, and overall thinking behind the scenes for the new minimum mathematics requirement, Math 108–Quantitative Reasoning. I will try to give an overview of the course in such a way that advisors, department chairs, and deans will better understand how Math 108 might play an important role in higher education at BYU-Idaho. This article will include a brief course description (to be found in the fall catalog), a historical summary of mathematics graduation requirements, a rationale that ties the course in with our institution's mission statement, concluding with a brief listing of course topics and answers to some frequently asked questions. At the end you will find sample problems from the four main components to the course, followed by an answer key.

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Course Description—Fall 2001 Catalog

(3.0 credits) Prerequisite: Math 101 with a grade of "B" or higher, or two years of high school algebra. You must also achieve a satisfactory score on the placement exam. The placement exam will cover topics from high school algebra I and II and will be offered during the first week of class.

Exploration of contemporary mathematical reasoning covering topics such as logic, problem solving, finance math, linear and exponential modeling, probability and statistics. Will satisfy both the BYU-Idaho and State of Idaho Core math requirements. Will not serve as a prerequisite for college algebra, trigonometry or any calculus-based courses.

Historical Overview of Math Requirement

Like a back-country hiker pausing to look over the trail covered, I'd like to briefly summarize where we have been to give us a little better perspective and appreciation for the upcoming change.

In the late 1970's, after significant effort in committee review, visitations to other campuses, the study of many other college catalogs, and other departmental work, a proposal was made at Ricks College to introduce a mathematics graduation requirement. Apparently, at that time only a handful of two-year colleges around the nation had a math requirement of any kind. In the fall of 1980, Math 100A–Arithmetic, became our minimum requirement for graduation.

By the end of the decade, however, during fall semester of 1989, the visiting Northwest Association of Schools and Colleges team challenged Ricks College to upgrade to a course that would be considered collegiate level. In response to that challenge, Math 101–Intermediate Algebra, was introduced in the fall of 1994 as the new mathematics graduation requirement. An ACT math score of 22 or better alternatively fulfilled the requirement. Throughout the remainder of the decade these two options remained in force.

Math 103–Problem Solving, was modified as of 1994 to meet the needs of our students who transferred to other Idaho institutions. It fulfilled the Idaho math core requirements and provided an alternative to Math 101 for a small niche of students. In the fall of 1998, our department voted to discontinue Math 103 and create a more rigorous quantitative reasoning course to be numbered Math 108, suggesting that it could be closer to a college algebra level of rigor but still approachable to general education students.

At the exit meeting on October 8, 1999, for the 10-year visitation of the Northwest Association of Schools and Colleges, it was recommended that, "There needs to be a review of what constitutes General Education classes—especially in regards to Math and Human Relations." As a math department we were grateful that we had already put together a solid quantitative reasoning course, one that was in fact a parallel course to many of the four-year schools in the intermountain region, more fully meeting accreditation standards.

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So Math 108 officially enters as the new minimum graduation requirement for specialized, associate, and baccalaureate degrees at BYU-Idaho for incoming freshmen as of fall semester 2001. The ACT math score will not serve as a waiver any more, and as a result, the majority of students will be taking either Math 108 or Math 110. Exceptions would include students with A.P. calculus credit, concurrent enrollment of college algebra, elementary education majors who take Math 305 and 306, and science or engineering majors who come in as freshmen enrolled for calculus. We are anticipating a significant increase in the number of Math 108 sections being offered, as well as an increase of teachers who will be teaching the course for the first time. We have had much dialogue within our department over the last year or so preparing for these changes, and we look forward to the challenge.

Rationale for Quantitative Reasoning

I have always felt that the first three components of our institution's mission statement have great bearing on what we are trying to accomplish in Math 108:

1. Build testimonies of the restored gospel of Jesus Christ and encourage living its principles.

2. Provide a quality education for students of diverse interests and abilities.

3. Prepare students for further education and employment, and for their roles as citizens and parents.

Galileo wrote centuries ago that "Mathematics is the alphabet with which God has written the universe." I often tell my math students that the Lord is the greatest mathematician of all and that if we are to be more like him then we will need to know mathematics. Every so often in the Romney hallways, during finals week, you will find a written comment or two from frustrated students such as, "We don't need to worry about mathematics after this life because we are supposed to create worlds *without* number." Generally speaking however, my students accept the basis of Galileo's statement and it sets a nice foundation for the rest of the semester.

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With regard to the second and third components of our mission statement, there is a great connection to the prospective audience and objectives of Math 108. We have a wide range of mathematical preparation on the part of our students. Some arrive with only a pre-algebra experience in high school; others come with Advanced Placement or even on-site college credit in calculus; the majority are somewhere between the two extremes. Returned missionaries also figure in here, having used little math during two years of knocking on doors around the world. We have extremely diverse mathematical abilities in our student body, combined with a wide variety of interests demonstrated by the number of students from different majors who request Math 108.

Through the evolution of our course design, Math 108 seems to now be focusing in on three main objectives for our students:

1. To prepare students for mathematics found in other *college* courses.

2. To help students develop quantitative reasoning skills necessary for a successful *career*.

3. To provide students with critical thinking and reasoning skills to deal with the major issues in *life*.

These three objectives parallel the very sense of the third component of the college's mission statement by preparing our students to become numerate citizens and parents, better equipped to deal with the quantitative complexities of a changing society. The current national dialogue over our potential ten-year budget surplus provides a classic example of how numeracy is vital to the democratic process. Such contemporary connections of Math 108 to the information-based world around us makes for a very rewarding and meaningful collegiate course.

The textbook that our math department has adopted originated from a quantitative reasoning project at the University of Colorado. Mathematics professor William Briggs teamed up with astronomer Jeff Bennett to write *Using and Understanding Mathematics: A Quantitative Reasoning Approach*. The Addison-Wesley second edition will be out for fall semester 2001.

In the instructor's forward to the text, the authors mention a pedagogical dichotomy in our approach to mathematics education: that of being content-driven versus context-driven. Content-driven textbooks organize the curriculum by mathematical ideas, and after the presentation of a given topic will provide a few examples of possible applications. Context-driven books, on the other hand, place mathematics in practical contexts, thereby having the applications drive the curriculum while presenting the mathematical ideas as they are needed within the applications themselves. Our approach to Math 108 fits under the latter category. The text is highly readable and should not be considered remedial. When algebraic steps are required of the student, an adequate review is provided to help the student brush up on previously learned skills.

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Some Key Skills of Quantitative Reasoning

To summarize the content of Math 108, I've listed its key ideas or skills, separated into four categories: Logic and Problem Solving, Quantitative Issues, Probability and Statistics, and Mathematical Modeling. These four divisions represent a common core that we will be teaching throughout the department. Typically, there is not enough time to teach all that we would like. A fifth and final component, Further Applications, which I have not listed in the table below, comes at the end of *Using and Understanding Mathematics*. Instructors may choose topics from these last chapters as appropriate and timely. Topics include integration of mathematics into art and architecture, fractal geometry, apportionment, theory of voting, network analysis, and so on.

Some Frequently Asked Questions About Math 108

- *What is the difference between Math 110–College Algebra, and Math 108–Quantitative Reasoning?* Math 110 is, in essence, a precalculus course, preparing science, math, and pre-professional majors for additional mathematics courses. Math 108, on the other hand, has a different objective. We intend for the course to apply to quantitative issues in our contemporary society (see Representative Problems below), and not as a prerequisite to higher mathematics. I used to

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Logic & Problem Solving	Quantitative Issues	Probability & Statistics	Mathematical Modeling
• fallacies	• common percentage abuses	• population v. sample	• linear v. exponential growth and decay
• arguments	• absolute v. relative change	• bias	• doubling time
• negations	• scientific notation	• controlled experiment	• half-life periods
• truth values	• significant digits	• placebo effect	• Rule of 70
• and vs. or	• principal v. interest	• blinding	• birth v. death rate
• conditionals	• simple v. compound	• margin of error	• carrying capacity
• truth tables	• APR v. APY	• confidence interval	• logistic growth
• logical equivalence	• lump sum v. annuities	• freq. tables, histograms	• overshoot and collapse theory
• converse	• liquidity, risk, return	• bar, line, pie charts	• logarithmic scale
• inverse	• cash, stocks, bonds	• correlation v. causation	• earthquake magnitude
• contrapositive	• total v. annual returns	• distribution	• decibels, pH
• Venn diagrams	• installment loans	• mean, median, mode	• modeling
• inductive v. deductive	• amortizations	• outliers	• domain v. range
• critical thinking	• equity	• range, quartiles	• periodic functions
• units	• credit danger	• 5-number summary	• rates of change
• conversion factors	• income tax brackets	• boxplots	• slope
• Polya's 4-step process of problem solving	• gross income	• standard deviation	• linear functions
	• adjusted gross income	• normal curve	• exponential functions
	• taxable income	• 68-95-99.7 Rule	• radioactive dating
	• FICA	• percentiles	
	• marriage tax penalty	• statistical significance	
	• tax-deferred savings	• hypothesis testing	
	• tax credit v. deduction	• outcomes, events, odds	
	• national deficit v. debt	• independent v. dependent events	
	• Social Security issues	• probability rules	
		• law of averages	
		• expected value	
		• gambler's fallacy	
		• permutations v. combinations	

tell students that it is a terminal math class, clarifying that it does not terminate in death. Now, however, I add that they can move into Math 221, Statistics, having had Math 108 (see below).

- *So will Math 108 suffice as a prerequisite to Math 221 Statistics?* About 25% of the Math 108 curriculum will deal with probability and statistics and in some ways will do a much better job of preparing them for an introductory statistics class than would college algebra. Key ideas from both descriptive and inferential statistics, which they will confront in Math 221, are discussed on a more conceptual level. Our main concern is that students in Math 221 have enough algebraic skills. The placement exam will help address this issue as we try to counsel those who are in need of some remediation in algebra.

- *How much mathematical background must a student have to be successful in Math 108?* Math 101, or Algebra II from high school, is the prerequisite. A placement test will be given the first week to help analyze their algebraic skills. We intend to have some help available in the form of videos, review packets, Math 108 web pages, and mini-workshops to brush up on their algebra at the first of the semester. They will be offered one retake on the placement exam. We will encourage our Math 108 teachers to counsel those students with low placement scores to pick up a refresher course in Math 101, Intermediate Algebra, or even Math 100B, Basic Algebra. Half the battle in collegiate mathematics is making sure you start out at the appropriate level. As many of us know, the curriculum in undergraduate mathematics is extremely sequential, and missing critical algebraic skills at one level will cause nothing but grief higher up the ladder.
- *Will Math 108 help prepare students for Math 110?* Algebraically, Math 108 will not help students prepare for Math 110. The proper preparation continues to be Math 101–Intermediate Algebra. Students should not take both 108 and 110. If they have plans of transferring to another university, they need to find out which course is required there. We realize there will be some students that will change their major later and have no choice but to take both. The additional perspective that they will have gained as they develop quantitative reasoning skills will nevertheless pay off for them eventually.

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Summary

I hope the commentary above will be beneficial to those who advise our students. Math 108 is a rewarding class both to teach and to take, due to its applicability to other college courses, careers, and life in general. Concepts are very approachable, the text is extremely readable and interesting, calculator technology brings a great pedagogical tool to the classroom, and best of all, the real curriculum of quantitative reasoning is all around us in our day-to-day contemporary society.

We envision Math 108 being not just a liberal arts alternative to college algebra but rather a very dynamic and applied mathematics course enhancing higher education at BYU-Idaho for a wide variety of majors. Combined with Statistics, Quantitative Reasoning makes a very solid "one-two punch" for a great number of our students. We look forward to your questions, comments and suggestions from across campus, helping us to promote a higher degree of numeracy for our undergraduate students.

Sample Problems from Math 108

Below are a few sample problems from each of the four categories in the Math 108 curriculum; answers follow. Should you disagree with or have an interest in the solutions, I would encourage you to audit the course, doing something perhaps many consider unthinkable (or an opportunity for integration)—take a mathematics course for the fun of it!

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Part 1: Logic and Problem Solving

1. Identify a potential fallacy in the following statement and justify your choice: "Following President Reagan's defense buildup, the Soviet Union began the process of democratization that ultimately led to its breakup. Therefore Reagan is responsible for the changes that led to the demise of the Soviet Union."

2. Since 1978, the government of China has officially allowed only one child per family. The stated goal of this policy is to reduce China's population from more than 1.1 billion today to about 700 million by 2050. One of many social consequences as a result has been the serious shortage of females in the young adult population. The reasons for this are fairly controversial and may include selective abortions and infanticide.

   Suppose China were to replace this one-child policy to a one-son policy. That is, assume each family were allowed as many children as desired until the first son is born at which point the limit is met. In theory, a family would be allowed to continue having children indefinitely until the first boy arrives. How would this policy impact the average number of children per family? Assume that the probability of having a boy is 50% and that multiple births are not figured into the problem.

3. Suppose that you would like to drive your race car around a 1-mile track for two laps with an average speed of 60 mph. (1 mile per minute). You drive the first lap (one mile) around the track at an average rate of 30 mph. How fast would you have to drive the 2nd lap to meet your overall goal of 60 mph average for the 2 miles?

Part 2: Quantitative Issues

4. Suppose you invest $1000 in the stock market Monday morning. By the end of the day your stocks lose 5% of their value. On Tuesday, they gain 5% of their value. Would you break even by the end of trading period on Tuesday? Explain.

5. Suppose that Jill at age 22 decides to invest $150 monthly into a portfolio that earns 10% compounded monthly and she does this until she turns 30 (8 years of investing). At age 30

she then stops contributing to her portfolio and lets the balance earn 10% until she retires at age 67 (37 years later). Now suppose that Mark (the same age as Jill) decides to unwisely procrastinate investing a portion of his income until age 30. At this point in his career, he decides to consistently invest $150 monthly (compounded monthly at 10%) until he retires at age 67 (37 years of investing). How much more money will Jill have than Mark when they both reach age 67 and how much of their own money (principal) was invested?

6. Assume the Robertsons purchased a home 10 years ago and set up their mortgage with a starting balance of $92,000 at 7.25% for 30 years. How much equity would the Robertsons have in their home presently if we assume they could sell the home at a fair market price of $145,000, they haven't paid any extra principal during the 10 years and the closing costs / realtor fees will amount to 7.25% of their selling price?

7. In the tax year 2000, Sid and Myra had adjusted gross incomes of $16,200 and $20,300, respectively. They had no dependents and they filed jointly as a married couple, claiming the standard deduction. Find the amount of their marriage penalty—that is, the amount of additional tax that they paid over what they would have paid if each had been single.

Part 3: Probability and Statistics

8. Consider a region that is prone to 100-year floods (that is, floods that have a 1% chance of occurring in any particular year). What are the chances that the region will experience at least one 100-year flood during the 21st century?

9. A television talk show host asks viewers to respond to a poll asking, "Should immigration be further restricted?" Viewers are given two 900 numbers to call: one for "yes" answers and one for "no" answers. Each call costs the viewer 50 cents. At the end of the show the host announces that 62% of Americans believe immigration should be restricted further. Comment on the validity of this conclusion.

10. Suppose you are interviewing for a new position at a high tech corporation and are told by the interviewer that the average annual salary is $37,000 at this company. Should you assume that half of the employees are earning under $37,000 and half are earning over that amount? Explain.

11. The school board in Boulder, Colorado, created a hubbub when it announced that 28% of the Boulder school children were reading "below grade level," and concluded that methods of teaching reading needed to be changed in a major way. The announcement was based on reading tests on which 28% of Boulder school children scored below the national average for their grade. Discuss the misconceptions that the school board might have and why 28% below reading level could be considered good news not bad.

Part 4: Modeling

12. Imagine a type of bacteria that divide every minute, as long as they have enough nutrients. Suppose that you place a single bacterium in a nutrient-filled bottle at 11:00 AM It grows and

divides into two bacteria at 11:01 and into 4 bacteria at 11:02. Now suppose that the colony of bacteria completely fill the bottle by 12:00 noon. At what time was the bottle half full?

13. According to a Time magazine article a few years back, some exponential modeling done by University of Washington sociologist, Dr. Rodney Stark, suggests that international membership of The Church of Jesus Christ of Latter-day Saints is growing at an annual rate of 4.9%. How long would it take for international membership to double based on that growth rate?

14. Assume that a new vehicle loses approximately 7% of its resale value each year. How much would a the car be valued at six years later if the original price were $18,500?

Answers to Sample Problems

1. False Cause or *Post Hoc* fallacy. Event A precedes event B doesn't necessarily mean A caused B. It could still be true but logic dictates that it doesn't have to be true. (This makes for a rich discussion of the relationship between logic, mathematics and politics.)

2. An average of two children per family. The best way to see this is to start with say 100,000 families and make a simple tree diagram with male and female branches working down the tree. Calculate the average at each level and look for a limit or trend.

3. Since to average 60 miles per hour for the two laps requires 2 minutes of time for the two miles and you have already spent the first two minutes on lap #1, you will not be able to drive fast enough during lap 2 to meet your average goal. Cannot be done.

4. You will have less money by the end of Tuesday. The 5% decrease takes your investment to $950 and the next day's increase of 5% brings your investment to $997.50.

5. Jill will end up with approximately $873,401 with this scenario having invested only $14,400 of her own money. Mark, on the other hand, will finish with approximately $698,974 having invested $66,600 of his own money. The first eight years that Jill spent investing had a much greater effect than the last 37 years of Mark's investment due to compounding a significant lump sum over a long time.

6. The Robertson's monthly payment would be $627.60 and after 10 years their payoff on the home would be about $79,406. After paying the closing costs, realtor fees and the original loan balance their equity is approximately $55,082.

7. Being married and filing jointly the tax would be $3,532.50 and filing as two single individuals it would be $3,315. The difference between the two constitutes the marriage penalty of $217.50.

8. About 63.4%. This can be found by finding the probability that there will not be a flood and finding the complement. 1 - (499/500)^100.

9. Since this is a typical voluntary response survey and not a random sampling of Americans it should be regarding with suspicion. It could have a high degree of bias and most likely provides few meaningful statistics.

10. If the average reported were a median then this assumption could be made. A mean, however, can be easily inflated due to high salaries of top executives in the company. The average would then not represent a halfway point in salaries.

11. Since national reading scores are often normal (bell-shape) distributions, an average district would have 50% of their students reading below grade level and 50% reading above. The fact that 28% of Boulder school children were reading below grade level is actually a sign that the district is well above average.

12. 11:59 A.M. The bacteria will then double one more time and fill the bottle at noon.

13. Using the Rule of 70 we approximate 14.3 years for doubling time. (70 ÷ 4.9 = 14.3.)

14. About $11,970. This is found by multiplying the original amount of $18,500 by (0.93)^6. The car is in other words, retaining 93% of its value each year.