PROVING PYTHAGORAS

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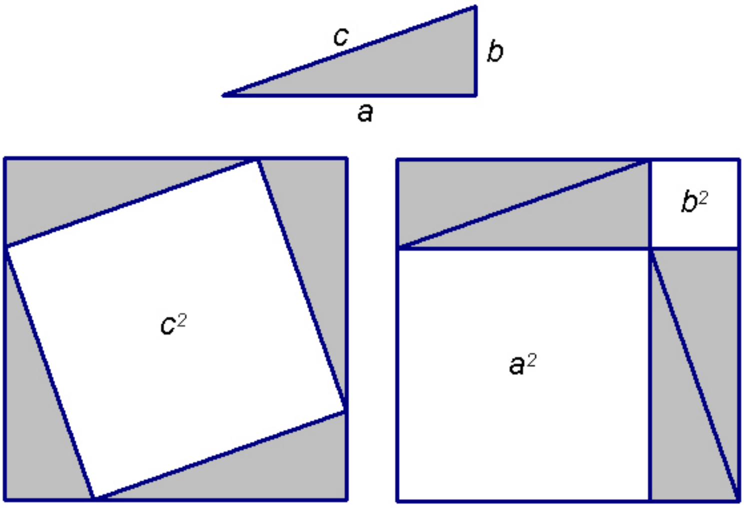
French mathematician Henri Poincaré said: "The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful."[1] Believing that mathematics is simply the unimaginative manipulation of numbers is akin to believing that music is just the tedious manipulation of notes. There is so much in mathematics that is creative, artistic, and even exquisite.

When mathematicians talk about 'beauty' in a proof or solution, they are praising its conceptual elegance rather than its visual appeal. As an example, consider the proof of the Pythagorean Theorem given in the figure below:



Both figures are the same size, since they are squares that measure a+b on a side. Because each figure includes four copies of the triangle (shaded gray), the white space in the figure on the left (c^2) must equal the white space in the figure on the right (a^2+b^2).

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Although there are hundreds of published proofs of the Pythagorean Theorem, this proof is perhaps one of the most elegant. Its brevity, simplicity, and transparency give it an aesthetic sophistication. An elegant proof is one that is unusually succinct or one that arrives at its conclusion in an impressive or unexpected way. It is one that uses the tools of mathematics effectively. Just as a landscaper does not choose a backhoe to dig a hole for a flower, a mathematician would prefer not to use a bulky mathematical tool if a simpler one will suffice.

In contrast to the striking proof above, consider the comparatively "ugly," proof of the "Four-Color Theorem." The theorem itself is simple. Suppose you are given a map, some colored pencils, and the goal of coloring the map with the least number of colors so that no two touching countries are the same color. The Four-Color Theorem states that four colors will suffice no matter how complicated the map is. Hundreds, if not thousands, of mathematicians have tried to come up with an aesthetically pleasing proof of this theorem, but the cleanest verification to date is clunky at best. Robin Thomas and his colleagues at the Georgia Institute of Technology in Atlanta proved the theorem by a "brute force." They reduced the total number of possible map configurations to 633 and checked these arrangements case by case using a computer. Although most of the mathematical world accepts their solution as a proof, no human could realistically check the details of the proof because it would take more than a lifetime. Their solution was valuable in that its implementation helped advance computer technology, but it was not neat or tidy, and it was certainly not elegant. Mathematicians continue to seek a more aesthetically pleasing solution.

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As G. H. Hardy noted: "The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics."[2]

Notes

1 J. H. Poincare (1854-1912), (cited in H. E. Huntley, *The Divine Proportion*, Dover, 1970).

2 G. H. Hardy (1877-1947), *A Mathematician's Apology*, Cambridge University Press, 1994.