Implementation of Orthotropic Elasto-Plastic model in MOOSE and Pellet-Cladding Interaction Simulations

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Abstract

Cladding is the first protection against fission products being released in a light-water-reactor (LWR). To avoid cladding failure, many simulations of accident conditions have been completed. These simulations accounted for pellet-cladding interaction, corrosion, elasticity and von Mises plasticity of the cladding, to name a few. The concern addressed in this work is that the isotropic von Mises plasticity model used in many of these simulations is insufficient for many anisotropic cladding materials. Zircaloy-2 is commonly used in LWRs and it is known to be highly orthotropic. Zircaloy-2, like many other hexagonal close-packed (HCP) metals and alloys, has displayed strong strength differential (SD) effect and some pressure sensitivity (PS) in its yield function. To model Zircaloy-2 or other HCP cladding materials, the general asymmetric yield function proposed by J. W. Yoon, Y. Lou, J. Yoon, and M. V. Glazoff [Int. J. Plast. 56, 184-202 (2014)], is implemented into Multiphysics Object-Oriented Simulation Environment (MOOSE) with associated and non-associated flow plasticity capabilities. The proposed model uses the stress tensor’s first invariant to model PS and the third deviatoric invariant to model SD effects in both tension and compression on the yield surface. The PS and SD effect is observed in the model after calibration from experimental data. The simulations of a UO$_2$ pellet expanding thermally in contact with a Zircaloy-2 cladding compare the proposed function with the von Mises model in accident conditions.

Introduction

Fuel rod cladding in nuclear reactors is the first precaution against fission product release. Much research has been done in regards to the dangerous causes of cladding failure. A general way to categorize the causes for failure (according to Groeschel et al. 2002) is: localized corrosion, pellet-cladding interaction, undetected manufacturing defects, and debris fretting. Pellet-cladding interaction accounts for approximately one out of eight cladding failures in boiling-water-reactors (BWR) from 2003 to 2006 (Dangouleme et al. 2010). Pellet-cladding interaction is the only cause of failure that will be investigated in this work. Simulations have been done with the von Mises yield function to account for the plasticity of the cladding during accident and failure conditions. However, the von Mises yield function only accounts for isotropic plasticity, and therefore, is insufficient for some cladding materials.

Light water reactors (LWRs) and BWRs typically use Zr-based alloys for fuel rod cladding. Zr base alloys are used as cladding because of their low neutron absorption cross-sectional area (meaning that it does not alter the nuclear reaction by
absorbing neutrons), high strength-density ratio, and corrosion resistance (C.R.F. Azevedo 2011). Zr-based alloy drawbacks include high price, a hexagonal closed packed (HCP) crystal structure, and orthotropic plastic behavior. Zircaloy-2 and Zircaloy-4 are the two most used Zr-based alloys for cladding. Both Zircaloy-2 and Zircaloy-4 display a noticeable strength differential (SD) effect. This SD effect is the different stresses needed to begin plastic deformation in tension and compression. The von Mises model cannot account for this difference in the tension and compression stresses, it predicts that the same stress needed to begin plastic deformation in tension is the same as in compression. Zircaloy-2 and Zircaloy-4 alloys also have some hydrostatic pressure sensitivity (PS) in their plastic behavior, which means that the stress needed to reach plasticity is different at different pressures. A strong orthotropic behavior in both alloys is evident in account of their texture developed during manufacturing. This orthotropic behavior is the different stresses needed to reach plasticity in different directions, e.g. $\hat{x}, \hat{y}, \hat{z}$. These stresses are the same in the von Mises model. Many models have been developed to accurately describe the plastic behavior of metals and research is still active for orthotropic yield functions for metals with anisotropic behavior.

**Basics of Plasticity and the von Mises Model**

Stress is force per unit area applied on an object. The force has a magnitude and a direction. The surface of the object has a normal direction. Stress is represented mathematically by combining these two directions and the magnitude of the force into a rank two tensor.

Strain is the change in the length of the object divided by the original or current length. Zero strain is equal to no dimensional change, whereas negative is compression and positive strain means tension. Elastic strain is changing the length of the object through stress but when the stress is removed it recovers the original shape. Plastic strain is the change of the length but after ceasing to apply a stress it does not go back to the original shape, it is deformed permanently.

Fig. 1 shows a typical strain and stress relationship. When the strain begins (loading) the stress grows linearly (the slope is referred as Young’s modulus) and at the point where it hits the yield strength it begins to change slope and from there the yield strength increases as the strain increases (hardening). If we stop applying strain (unloading) then the material will undergo elastic contraction and undo all the elastic strain but the plastic strain will stay as a permanent deformation (Krabbenhoft 2002). In terms of energy only the energy used in elastic strain can be recovered. The energy used in plastic strain will go to permanent deformation.
The increase of the yield strength is referred as “hardening” and it depends on all the internal factors of the material under stress. All these internal factors can be grouped together empirically in one internal variable named \( q \) and the hardening is directly calculated from this internal variable.

If we were to continue applying a strain the material would eventually fracture. The proposed model by J. W. Yoon and his group (Yoon et al. 2014) only works until the point before fracture and it doesn’t provide the point where fracture begins. The usefulness of the model is to find the stress-strain response up until ultimate tensile strength.

![Figure 1: Typical strain and stress relationship](image)

The stress will be represented as the rank two tensor \( \sigma \) as follows:

\[
\sigma = \begin{bmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{bmatrix}, \quad xy = yx, \quad xz = zx, \quad yz = zy
\]

The diagonal terms in the stress tensor represent the tension or compression of the normal to the surface and the off-diagonal are the shear stresses. For example if
we put stress on the surface that has a normal of \( \hat{x} \), the stress could be any of the top three entries in the tensor \( xx, xy, xz \). If the stress is only in the \( \hat{x} \) direction it means that we are either putting tension or compression. Tension is represented by a positive value in any of the diagonal entries of the stress tensor while compression is represented by a negative value in the same diagonal. If the \( \hat{x} \) surface direction has a stress in the \( \hat{y} \) direction it will be shear stress. A rank four tensor has \( 3^4 \) entries and is represented with four underlines while two are used for rank two tensors.

The von Mises model is the most commonly used plastic yield function and its mathematical form is:

\[
f_{\sigma,q} = \sqrt{3J_2(\sigma) - \sigma_y(q)} = 0
\]

\( J_2 \) is the second deviatoric (deviatoric represents the shear part of the stress tensor) invariant (meaning, it does not change with different basis) of \( \sigma \). \( \sigma_y(q) \) is the yield strength and is a function of \( q \). The invariants of tensors don’t change if \( \hat{x}, \hat{y}, \) and \( \hat{z} \) are switched as long as they still are orthogonal and the shear is changed appropriately. The formula for \( J_2 \) is

\[
J_2 = \frac{1}{6}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2] + \sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2.
\]

The von Mises model assumes that the material under stress does not have plasticity under pure hydrostatic stress. The model also assumes that there is no difference
if the material is in compression or in tension. The yield surface is the surface that represents the points in stress direction at which plasticity begins, Fig. 2 represents the von Mises yield surface. Fig. 2 shows that the shape is a cylinder and an open surface. According to this surface, if the strain were to increase at the same rate in all three directions (pure hydrostatic) the material would expand elastically to infinity without ever touching the yield surface. Other models developed to display more anisotropic plastic behavior include Hill 1948; Barlat and Lian 1989; Barlat et al. 1991, 1997, 2005; Banabic et al. 2005; Yoshida et al. 2013; among many. They include more anisotropic coefficients and invariants to increase their capacity to display different plastic anisotropic behaviors in their yield surface. From their work in aluminum and steel alloys, Spitzig and Richmond (1984) added a coefficient and the first invariant of the stress tensor to the von Mises model to account for PS plastic behavior:

\[ f_\sigma = \sqrt{3J_2} + aI_1 \]

\( I \) is the trace of the stress tensor and it is related to the PS effect. \( a \) is a constant. Later on Cazacu and Barlat (2004) modified the yield function proposed by Drucker (1949) to display SD effect using the third stress deviatoric invariant:

\[ f_\sigma = J_2^{3/2} - cJ_3 \]

\( J_3 \) is the determinant of the stress tensor and \( c \) is a constant. These last two models can only account for PS or SD effect. The general asymmetric yield function, proposed by Yoon et al. (2014) was selected as an addition to MOOSE developed by the Idaho National Laboratory (Gaston et al. 2009) due to its capacity to model all three SD, PS, and orthotropic plastic behaviors at once or separately.

**The Proposed General Asymmetric Yield Function**

Orthotropic behavior results from the HCP lattice structure. HCP lattice structure with slip planes are shown in Fig. 3. Though a single crystal with HCP lattice has orthotropic plastic response, a random distribution of grains in a polycrystalline microstructure can nullify the effect and render an isotropic behavior. However, metal forming processes, such as cold rolling, can realign the initially randomly oriented grains to preferred orientations (texture) and result in orthotropic behavior.
Figure 3: HCP lattice structure with different slip planes (Hasija et al. 2003).

The proposed model is:

\[ f(\sigma) = b I_1 + (J_2^{3/2} - J_3^{''})^{1/3} \]

The coefficient \( b \) controls the PS behavior represented by \( I_1 \). \( J_2' \) and \( J_3'' \) are the second and third invariants of the transformed stress tensor \( s' \) and \( s'' \) respectively. Both \( s' \) and \( s'' \) are calculated from the stress tensor in this manner:

\[ s' = L' \sigma, \quad s'' = L'' \sigma \]

where

\[ \sigma^T = [\sigma_{11} \quad \sigma_{22} \quad \sigma_{33} \quad \sigma_{23} \quad \sigma_{13} \quad \sigma_{12}] \]

\[ L' = \begin{bmatrix} \frac{(c_2' + c_3')}{3} & -c_3'/3 & -c_2'/3 & 0 & 0 & 0 \\ -c_3'/3 & \frac{(c_2' + c_1')}{3} & -c_1'/3 & 0 & 0 & 0 \\ -c_2'/3 & -c_1'/3 & \frac{(c_1' + c_2')}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5' & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6' \end{bmatrix} \]
\[ L'' = \begin{bmatrix}
   (c''_2 + c''_3)/3 & -c''_3/3 & -c''_2/3 & 0 & 0 & 0 \\
   -c''_3/3 & (c''_3 + c''_5)/3 & -c''_1/3 & 0 & 0 & 0 \\
   -c''_2/3 & -c''_1/3 & (c''_1 + c''_2)/3 & 0 & 0 & 0 \\
   0 & 0 & 0 & c''_4 & 0 & 0 \\
   0 & 0 & 0 & 0 & c''_5 & 0 \\
   0 & 0 & 0 & 0 & 0 & c''_6
\]  

The coefficients set up in \( L' \) and \( L'' \) are explained by (Barlat et al. 1991). To get the same result as the von Mises model, set \( c'_1 = c'_2 = c'_3 = c'_4 = c'_5 = c'_6 = \sqrt{3} \) and \( c''_1 = c''_2 = c''_3 = c''_4 = c''_5 = c''_6 = b = 0 \). Fig. 4 has the mentioned constants to reduce the proposed model to the von Mises model with the exception of different values for \( b \) to show the PS behavior in the yield surface. Fig. 4 shows that the PS behavior turns the cylindrical von Mises surface into a conic-like shape. The tip of the conic shape depends on the sign of \( b \). If negative, then the tip is in the space with all three diagonal stresses negative. The opposite is true when positive. Only the tip predicts plasticity when there is even pressure in all directions. This tip may never be reached because materials usually fail before that point. The coefficient \( b \) is typically calibrated through uniaxial tests in a hydrostatic pressure chamber (Yoon et al. 2014).

The coefficients of \( L' \) govern the orthotropic behavior. Setting \( c''_1 = c''_2 = c''_3 = c''_4 = c''_5 = c''_6 = \sqrt{3} \) but changing \( c'_1, c'_2, c'_3 \) causes differences for the three orthogonal directions in the yield surface. This is shown in Fig. 5. In the case of Fig. 5a the stress in the \( \hat{x} \) direction necessary to reach the yield surface is much greater than the other two directions. Even the difference between the \( \hat{y} \) and \( \hat{z} \) direction is visible. There is no tension-compression asymmetry so no SD effect is visible. The shape is also cylindrical and not a cone indicating there is no PS behavior. Only orthotropic behavior is changed by \( L' \).

The coefficients of \( L'' \) establish the SD effect. Fig. 6 keeps \( c''_1 = c''_2 = c''_3 = c''_4 = c''_5 = c''_6 = \sqrt{3} \) and \( b = 0 \) to display the SD effect. Both yield surfaces displayed strong SD effects in the tension-compression asymmetry. The yield surfaces displayed no PS behavior; however, it was hard to determine if there was some orthotropic behavior because of the SD effect.

Because of the model flexibility to display PS, SD, and orthotropic behavior separately and the ability to revert back to earlier models, namely von Mises; Spitzig and Richmond [1984]; and Cazacu and Barlat [2004], the model proposed by Yoon et al. (2014) was implemented into MOOSE.
Figure 4: Cone-like shape of the yield surface in the principal stress space as an effect of the coefficient $b$, (a) $b = -0.2$ and (b) $b = 0.2$
Figure 5: Cylinder with oval opening of the yield surface in the principal stress space as an effect of the coefficients $c'_1, c'_2, c'_3$, (a) $c'_1 = -1, c'_2 = 3.5, c'_3 = -1.6$ and (b) $c'_1 = .5, c'_2 = .5, c'_3 = 2.0$
Figure 6: Cylinder with oval opening of the yield surface in the principal stress space as an effect of the coefficients $c''_1, c''_2, c''_3$, (a) $c''_1 = -2, c''_2 = 1, c''_3 = 3$ and (b) $c''_1 = 1.5, c''_2 = -0.8, c''_3 = -3$.
Implementation in MOOSE

MOOSE is a parallel finite element framework developed to solve implicitly tightly coupled equations (Gaston et al 2009). The proposed model is applied in MOOSE through the Backward Euler return mapping scheme (Belytschko et al. 2006). It is a fully implicit scheme. The scheme takes an initial elastic strain $\mathbf{e}_n$, plastic strain $\mathbf{e}_p$, and internal variable $\mathbf{q}$ and after a time step it returns $(\mathbf{e}_{n+1}, \mathbf{e}_{p+1}, \mathbf{q}_{n+1})$. During this process, the Newton-Raphson method is used to find the root $f(\mathbf{q}) = bI_1 + (J_2^{3/2} - J_3'^{1/2}) - \sigma_y(q) = 0$. The Newton-Raphson method requires the first and second tensor derivative of the equation $f(q)$ with respect to stress tensor to find when it is equal to 0. Part of the Backward Euler return mapping scheme calculates $\mathbf{q}$ from the plastic rate parameter and the second tensor derivative of $f(q)$. To allow the magnitude of the growth of $\mathbf{q}$ to depend on the plastic rate parameter and not in the second tensor derivative, the first tensor derivative was normalized and then the tensor derivative was taken to get the second tensor derivative. The full explanation of the scheme can be found in Belytschko et al. (2006). As stated before, a rank four tensor has $3^4$ entries and is represented with four underlines while the rank two tensor has two. The symbols : and $\otimes$ are, respectively, inner and outer products between tensors. For simplicity, in the following formulas $A = \frac{\partial f}{\partial S_{kl}}$ and $B = \frac{\partial f}{\partial S_{kl}}$.

The first derivative in tensor notation is:

$$\frac{\partial \phi}{\partial \sigma} = \frac{\partial \phi}{\partial I_1} \frac{\partial I_1}{\partial \sigma} + \frac{\partial \phi}{\partial J_2} \left( \frac{\partial J_2'}{\partial I_1} : A \right) + \frac{\partial \phi}{\partial J_3} \left( \frac{\partial J_3''}{\partial I_1} : B \right)$$

The formulas for the parts in the first derivative are:

$$\frac{\partial \phi}{\partial I_1} = b, \quad \frac{\partial I_1}{\partial \sigma} = \delta_{ij}, \quad \frac{\partial \phi}{\partial J_2'} = \frac{\sqrt{J_2'}}{2(J_2^{3/2} - J_3'^{1/2})^{3/2}}, \quad \frac{\partial \phi}{\partial J_3''} = -\frac{1}{3(J_2^{3/2} - J_3'^{1/2})^{3/2}}$$

$$A = S_{ij}(1 - \delta_{kl}) - (I_1 - S_{kl})\delta_{kl}$$

$$B = \left( I_1 - S_{kl} - S_{mn}(1 - \delta_{mn}) \right) \delta_{kl}(1 - \delta_{km}\delta_{kn}) + \left( S_{km}S_{lm}\delta_{kl}(1 - \delta_{km}\delta_{lm}) - S_{kl}S_{mn}(1 - \delta_{km}\delta_{lm}) \right)$$

The normalization, setting $E = \frac{\partial \phi}{\partial \sigma}$, becomes:

$$C = \frac{E}{\sqrt{E^2}}$$

The second derivative does not have $\frac{\partial \phi}{\partial I_1} \frac{\partial I_1}{\partial \sigma}$ because after the first derivative it becomes a constant. For simplicity, $f_{J_2} = \frac{\partial \phi}{\partial J_2}$ and $f_{J_3} = \frac{\partial \phi}{\partial J_3}$. The second derivative is a fourth order tensor formed by:
$$\frac{\partial C}{\partial \sigma} = \frac{1}{\sqrt{r \cdot \sigma}} \frac{\partial r}{\partial \sigma} - \frac{r}{(r \cdot \sigma)^{3/2}} \otimes \left( \iota^r \cdot \frac{\partial r}{\partial \sigma} \right)$$

with

$$\frac{\partial r}{\partial \sigma} = \frac{\partial f_{J_2}^r}{\partial J_2} \left( \iota^r : A \right) \otimes \left( \iota^r : A \right) + \frac{\partial f_{J_2}^{r''}}{\partial J_3} \left( \iota^{r''} : B \right) \otimes \left( \iota^{r''} : A \right)$$

\[+ \frac{\partial f_{J_3}^{r''}}{\partial J_2} \left( \iota^r : A \right) \otimes \left( \iota^{r''} : B \right) + \frac{\partial f_{J_3}^{r''}}{\partial J_3} \left( \iota^{r''} : B \right) \otimes \left( \iota^{r''} : A \right) \]

with the parts being

$$\frac{\partial f_{J_2}^r}{\partial J_2} = \frac{J_2^{1/2}}{4(J_2^{3/2} - cJ_3)^{2/3}}, \quad \frac{\partial f_{J_2}^{r''}}{\partial J_2} = \frac{J_2^{1/2}}{3(J_2^{3/2} - J_3^{''})^{5/3}}$$

$$\frac{\partial f_{J_3}^{r''}}{\partial J_3} = \frac{J_3^{1/2}}{3(J_2^{3/2} - J_3^{''})^{5/3}}, \quad \frac{\partial f_{J_3}^{r''}}{\partial J_3} = -\frac{2}{9(J_2^{3/2} - J_3^{''})^{5/3}}$$

After successful implementation in MOOSE, the model was calibrated to appropriate coefficients for Zircaloy-2 from experimental data.

### Calibration

The proposed model contains 13 parameters. The parameters $c_1', c_2', c_3', c_4', c_5', c_6'$ control the orthotropic and SD behavior in the principal stress space, on the other hand $c_4', c_5', c_6', c_4'', c_5''$ and $c_6''$ control the shear. The method of calibration was proposed by Yoon et al (2014) however the down-hill simplex method proved sufficient for the calibration of $b, c_1, c_2, c_3, c_1', c_2'$ and $c_3'$ from experimental data. The data is taken from the work of Ballinger and Pelloux (1981) in uniaxial and estimated bi-axial tests in JA cold worked and KA cold worked Zircaloy-2 sheets. From their data, the material shows strong anisotropic behavior. The $\sigma_y$ is 550 MPa. The coefficients are:

<table>
<thead>
<tr>
<th>Zircaloy-2</th>
<th>$b$</th>
<th>$c_1'$</th>
<th>$c_2'$</th>
<th>$c_3'$</th>
<th>$c_1''$</th>
<th>$c_2''$</th>
<th>$c_3''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>JA cold worked</td>
<td>-0.0060</td>
<td>1.1865</td>
<td>1.8013</td>
<td>2.0137</td>
<td>-1.9688</td>
<td>-0.5156</td>
<td>2.2656</td>
</tr>
<tr>
<td>KA cold worked</td>
<td>0.0012</td>
<td>1.1875</td>
<td>1.4375</td>
<td>2.4590</td>
<td>-1.0020</td>
<td>-0.9697</td>
<td>2.2422</td>
</tr>
</tbody>
</table>

The results for JA cold worked Zircaloy-2 are in the following graphs:
Figure 7: The yield function fit for JA cold worked Zircaloy-2 in (a) longitudinal ($\sigma_{xx}$) versus transverse ($\sigma_{yy}$), (b) longitudinal versus normal ($\sigma_{zz}$) and (c) transverse versus normal. The triangles are the experimental data. The dots are the fit.

During their work, Ballinger and Pelloux (1981) ignored the PS effect in their fit. The low value of the parameter $b$ in the Yoon et al (2014) proposed model sustains Ballinger and Pelloux claim that PS effect can be ignored for the JA cold worked...
Zircaloy-2 sheet.

The material hardening was also fitted to the power rule hardening formula:

\[ \sigma = \sigma_y \left( \frac{q}{\epsilon_0} + 1 \right)^n \]

Where \( q \) is the internal variable calculated in the Euler Backward scheme. Both \( \epsilon_0 \) and \( n \) are the parameters to be fitted. The fit was also performed with the down-hill simplex method. The associative flow rule was used for the calculation of \( q \).

**Associative versus Non-Associative Flow Rule**

As explained in the Implementation to MOOSE section, part of the Backward Euler return mapping scheme calculates \( q \) from the plastic rate parameter and the second tensor derivative of \( f_{\sigma,q} \). This part of the scheme uses the plastic flow rule to develop \( q \) every time step (Belytschko et al. 2006). This plastic flow rule calculates the direction of the flow of the plasticity by using the second tensor derivative of the yield surface. The associative flow rule relies on the normal to the surface of the proposed model to calculate the plastic flow and \( q \). The non-associative flow rule uses the von Mises’ surface for said calculations. Given that the von Mises’ surface is simpler, a cylinder with constant radius (see Fig. 2), compared to the proposed model (see Fig. 4, 5 and 6) the convergence of the Euler Backward scheme is faster and an approximation; the normal to the real surface of the proposed model will yield more accurate growth of \( q \) at the cost of more computing time. Fig. 8 shows the hardening difference in the JA cold worked Zircaloy-2 sheet case.
Figure 8: The hardening for JA cold worked Zircaloy-2 in (a) longitudinal, (b) transverse and (c) normal.

**Pellet-Cladding Simulation**

The simulation involves the effect of missing pellet surface (MPS). MPS is a manufacturing defect known to be one of the main causes of clad failure (Potts, 1997). The simulation geometry is seen in Fig. 9.
Only a quarter of the of the whole pellet and cladding cylinder is simulated. The longitudinal direction was aligned to the cylinder axis ($\hat{y}$), normal direction to $\hat{x}$ and transverse to $\hat{z}$ in this simulation. A power ramp is simulated in the pellet through an increase in temperature of 1000k in a second. The thermal expansion will create the contact between pellet and cladding. The point of interest is the area where the MPS is in contact with the cladding. The largest plastic behavior is assumed to be there. All sides of the pellet not in contact with the cladding are fixed except the top. The top is allowed to move freely. Fixing the top causes the pellet to bulge against the cladding. In reality the pellet would not bulge as extremely as when the top of the pellet is fixed and it would not move as freely when is not fixed, the real behavior would be somewhere between the two extremes. Given that we are only interested in the difference between the von Mises model and the proposed model plastic behavior in the area around the PMS, the free top will suffice. The differences of $q$ and the three diagonal stresses in $\sigma$ are in Fig. 10 and 11.
Figure 10: Results of $q$ (intnl), only showing the cladding, after power ramp for 1 second in (a) von Mises model, (b) proposed model, (c) von Mises model and (d) proposed model.
Figure 11: Results of $\sigma$ after power ramp for 1 second in (a) $\sigma_{xx}$, (b) $\sigma_{yy}$ and (c) $\sigma_{zz}$ for the von Mises model and (d) $\sigma_{xx}$, (e) $\sigma_{yy}$ and (f) $\sigma_{zz}$ for the proposed model.
There are important differences between the results of the von Mises model and the proposed model. It is shown in Fig. 10 that $q$ is larger in the proposed model in many areas, specially in the back and center of the cladding where $q$ is more than 40% greater and it is a little shifted toward the $\hat{z}$ direction. This shift is comes as a result of the orthotropy of the material because of the slightly lower yield strength in the $\hat{z}$ direction compared to the yield strength in the $\hat{x}$. The orthotropy is more apparent in Fig. 11, the maximum tension in von Mises $\sigma_{xx}$ and $\sigma_{zz}$ are close to each other when compared to the maximum tension of the proposed model $\sigma_{xx}$ and $\sigma_{zz}$. Also the higher compression and lower tension in Fig. 11d compared to Fig. 11f show the presence of SD effects. Overall, $q$ and $\sigma$ results are significantly higher for the model proposed by Yoon et al. (2014) than the von Mises model.

Figure 12: Comparison between von Mises and orthotropic model for (a) $q$, (b) $\sigma_{xx}$, (c) $\sigma_{yy}$ and (d) $\sigma_{zz}$.
Conclusion

The model proposed by Yoon et al. (2014) was selected as a addition to MOOSE due to its capacity to model SD, PS, and orthotropic plastic behaviors separately and the ability to revert back to earlier models (namely von Mises; Spitzig and Richmond [1984]; and Cazacu and Barlat [2004]) that do not use all three behaviors. The model does require data from multiple uniaxial and biaxial tests for calibration that can be difficult to perform or obtain from current publications. The associative flow rule predicts the plastic flow better than the non-associative flow rule but it takes more computing time.

The proposed model showed an overall higher predictions in $q$ and $\sigma$ compared to the von Mises model. This is shown in Fig. 12. These higher predictions could affect significantly the overall behavior of the cladding and should be accounted for in cladding performance simulations. As MOOSE develops creep and fracture models to be used in conjunction with the proposed yield function, more accurate simulations should be performed.

Future Work

The proposed model’s variables are only calibrated for the temperature at which the uniaxial and biaxial tests were performed. Yield strengths of metals can change dramatically with large temperature changes (Antoun 2004). A temperature dependency should improve the effective range of the model but it would require more data to calibrate. A proper uncertainty study through Monte Carlo simulations should also be done in the future.
References


reactors,” Proceedings of 2010 LWR Fuel Performance, Orlando, FL.


