Calculus – Twice Flipped

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A physics teacher once asked me, "What are you guys in the math department teaching over there in Calculus? My students need to do simple derivatives and integrals by week three and they're still doing limits!" I wanted to spread out the cognitive load in Calculus 1—which is extremely high—and I wanted to find a way to get to derivatives and integrals faster, so I divided my calculus course objectives into the HOW's and the WHY's. In the HOW's, we learn the mechanics of derivatives and integrals along with simple word problem types. In the WHY's, we learn the mathematics behind those techniques. The structure of the class is helical so that topics are revisited multiple times for deeper understanding. In addition, I flipped the classroom so that the students prepare mundane tasks before they come to class, and we use the class period to delve more deeply into the topic. I would like to share how I do it and report the classroom research I did to answer the question: "How did my students in Calculus 1 perform in the subsequent course compared to the rest of the Calculus 1 students on campus in the last 5 years?" The results were significant. Hence, we have Calculus – Twice Flipped.

I tried to reverse-design the course based on long and short-term objectives. I hope that in five years the students will remember that a derivative is an instantaneous rate of change and a definite integral is a sum of infinitely many small pieces. In addition to the content goals for the course, I wanted to design the course to meet the following objectives: to reduce the cognitive load in Calculus 1, to get to derivatives and integrals faster, to get algebra skills up to speed quickly, to practice more word problems, to make passing the course impossible without some word problems skill, to encourage students to discover formulas and big ideas, and to do proofs in class with student participation and understanding.

The HOW's teach the techniques of Calculus from the perspective of the different function families: mechanics of taking the derivatives; linearization, optimization, and related rates applications; the mechanics of integration; total accumulation, average value, area between curve, and initial value applications. The structure is helical and allows students to revisit the same Calculus notions five times in different contexts (five different function families: power functions, exponential and logarithmic functions, trig and inverse trig functions, and hyperbolic functions).

The WHY's go back and teach the mathematics of the big Calculus ideas: limits, derivatives using the limit definition and graphical analysis, definite integrals and Riemann sums, volumes of solids, arc length, work, and exponential growth. The mathematical foundations of the techniques the students learned in the HOW's is taught along with proofs of important theorems.

The daily plan for a typical class period has three main components. They are a prepare activity, an in-class activity, and an online homework assignment. Prepare activities with an online quiz are due before the class starts. Some activities are discovery-oriented and some are just-in-time algebra reviews pertinent to the lesson.

I have found that making useful prepare activities, beyond just reading the text and answering some questions, involves at least some of the following suggestions. Lay out each class period in the semester so that you have a schedule for the topic(s) explored on any particular day. Make any reading assignments or video links that would be

useful to help students understand surface-level vocabulary and / or rote exercises. Imagine student questions and confusions that are frequently encountered on those topics. Devise an activity that will allow the students to do something on their own to discover concepts, that will challenge commonly-held beliefs, or that will review a topic needed for that day's lesson. Provide support material in addition to reading assignments such as video links, games, handouts, etc. Be aware that if you don't attach a quiz worth points (not just completion points), the students will blow off your prepare assignments and will miss a vital component of the learning experience. Also, do not go over prepare assignments in class. If you habitually go over the prepare assignment in class, the students have no reason to do it before class. However, I always make reference to the prepare assignment in class so that the students realize that it would have been valuable to have done it.

For example, to start off my lesson on the derivative rules for exponential and logarithmic functions, I give students a discovery activity in which they look for patterns in data to discover and verbalize the rules. At the beginning of class I ask the students, "So, when you found the slopes of the various points in the prepare assignment, did you see any patterns? What did you see?… Well, that's pretty amazing!" This activity is near the beginning of the semester so most students have done it and are excited to share their discoveries. In this way, the students will have discovered the formula, will hopefully remember it longer, and have not needed a formal proof to find it.

As an example of a review activity, I give students a worksheet which reminds them about absolute value notation and what it means in preparation for epsilon / delta proofs of limits. I then ask them to interpret the symbols in a different way. For clarity in the lesson that day, the students need to see some old notation from a new perspective. To start off this lesson, I would reference the prepare assignment by writing an interval expressed with an absolute value on the board and mention, "In the prepare assignment, you learned that this expression represents something more than just an absolute value expression. How would you describe this notation without using the words, 'absolute value'?" This is one of the questions on the quiz. I wouldn't go on until someone had answered the question correctly. If no one can come up with it, I would encourage the students to look it up in their prepare assignment in I-Learn.

In-class activities are intended to be done in groups at the boards. I don't choose groups, but have the students work with students nearby, 3-4 per group. They seem to get pretty well-mixed by chance. I don't grade any group work or in-class activities. Students are given a worksheet to work on together in groups at the boards. My TA and I go from group to group answering questions, giving guidance and hints, and giving lots of praise and encouragement. The worksheets are to be done on the board in groups so they can revisit the problems individually before they start their homework. Complete keys are provided in I-Learn for each activity. The idea is to explore the problems as a group, then re-explore them as individuals before they start their

homework, thus providing an opportunity for the students to re-boot their brains before diving directly into the homework. It is of great importance to use the language of Mathematics, in this case, and have them practice it as they work together in their groups.

Homework assignments consist of online homework completed every day before the next class period starts and SEEIT writing assignments. I give the students 10 online problems for homework each day. All homework assignments should be worth points (not completion points) or the students won't do them. Homework is due before class the day after it is assigned. Since this is a fast-moving daily class, the students need motivation to do their homework every night because I will build upon what we did in the next class period. I don't make a habit of going over homework problems in class unless many students have asked about a particular problem. Students can get help from any of the following: me, the TA, online resources, the Math Lab in the library, MyMathLab, etc.

SEEIT activities are writing assignments to help students review and coalesce the notions involved in a particular topic. The acronym stands for State, Elaborate, Exemplify, Illustrate, and Teach. SEEIT provides a framework for students to organize their ideas and gives them practice writing. Students write SEEIT assignments about each of the big topics in Calculus for a total of four writing assignments. They are fairly easy to grade (for content and correctness, not completion points) according to a rubric.

The helical structure of the course is demonstrated in both the HOW's and the WHY's. Topics are rearranged by function families rather than Calculus notions. The students discover the derivative rule for power functions, then practice various application problems for derivatives. They discover the anti-differentiation rules for power functions, then practice certain application problems of definite and indefinite integrals. Then they repeat the process with exponential and log functions. And repeat it with trig functions. And repeat it with inverse trig functions. And repeat it with hyperbolic functions. The Calculus ideas get repeated over and over so the students can continue to grow in their understanding.

After the mechanics of derivatives and integrals are studied, we tackle limits, first intuitively and then formally. We use limits to formally define derivatives and prove all the rules we learned mathematically. We use limits to formally define definite integrals and prove how the formulas we used in integration applications really work. We tie all that we have learned and used together by proving the important theorems.

In 2013, a year after I started teaching this course, I compared the grades of the students in three of my Calculus classes before I started flipping the course with the three I had just taught by the new method. I wanted to know for sure that I was doing no harm. The results were not significant, but indicated that the students were doing at least as well as in previous semesters and maybe a little better.

In 2017, after having taught the course over 15 semesters, I did a more extensive study. I compared all the grades of my students in the subsequent course over the last 5 years with the grades of all other Calculus students at BYU-Idaho in the subsequent course over the same time period. The two subsequent courses I tested were MATH 113, Calculus II, and MATH 215, Calculus II for Engineers. In both cases the average grade in the subsequent class was higher among my students and it was significantly higher ($p = .03$) among my students in MATH 215 than students of all other Calculus classes at BYU-Idaho.

I see many advantages that have come from flipping both the topics and the class.

- Students examine a topic superficially using a prepare activity.
- They explore that topic again in a group setting where they can share ideas and help each other using the language.
- They see that topic again as they try to master the problems individually that they have already seen in their groups.
- They apply that topic again as they do homework.
- Students are more mathematically mature by the middle of the semester when we tackle the mathematics of Calculus rather than just the techniques.
- Students taking Calculus I concurrently with Physics, Statics, etc., are able to do derivatives and integrals of simple functions within 3 weeks.

The idea is to explore the problems as a group, then re-explore them as individuals before they start their homework.

- Students have about 7 weeks to review algebra skills without having to process abstract notations of Calculus.
- Students get practice looking for patterns and get excited when they see them.
- Word problems make up 35% of exams. Students see word problems of the same type on exams at least six times.
- Students can understand and do epsilon / delta limit proofs of linear and non-linear functions.
- Students understand the difference between average and instantaneous rates of change and actually understand the limit of the difference quotient.
- Students understand Riemann sums and the definition of the definite integral.
- Students understand that integrals are adding up little pieces of something and can concentrate on applications without worrying about how to solve the integral.
- Students can follow and understand when we prove theorems in class. Together we prove the Mean Value Theorem, the Fundamental Theorem of Calculus parts I and II, and though they wouldn't have been able to come up with the proofs on their own, they are impressed with the techniques. Previously, they tuned me out completely.
- They appear to do better in the subsequent class.

Flipping the class and the course content has made an impressive change in the way I teach Calculus and in the way my students learn Calculus. It has been and continues to be a very rewarding experience.